SME5802

Introdução à Mecânica dos Fluidos Computacional
Gustavo Carlos Buscaglia

ICMC - Ramal 8176, gustavo.buscaglia@gmail.com

Project:

The purpose is to numerically study microfluidic manipulation of particles. The underlying ideas are described in the two articles "Arbitrary steering of multiple particles independently in an electro-osmotically driven microfluidic system" (S. Chaudhary and B. Shapiro, IEEE Trans. Control Systems Technol., 14:669-680, 2006) and "Control of microfluidic systems: Two examples, results, and challenges" (M. Armani et al, Int. J. Robust and Nonlinear Control, 15:785-803, 2005).

The physical setting is the narrow space between two parallel walls, in which electro-osmotic flow is induced by applying voltage to a number of electrodes (actuators). Electro-osmotic flow obeys the Stokes equations

$$-\mu \nabla^2 \mathbf{u} + \nabla p = 0 \ , \qquad \nabla \cdot \mathbf{u} = 0 \qquad \text{in}\, \Omega \tag{1}$$

with boundary condition

$$\mathbf{u}_\tau = m_{eo}\, \mathbf{E}_\tau \qquad \text{on}\, \partial\Omega \ . \tag{2}$$

Above, the subindex τ refers to the **tangential** component of a vector field, i.e.,

$$\mathbf{u}_\tau = \mathbf{u} - (\mathbf{u} \cdot \check{\mathbf{n}})\, \check{\mathbf{n}} \ . \tag{3}$$

The articles consider a situation in which all the surfaces in contact with the fluid have exactly the same properties, in which case it is not necessary to actually solve the Stokes equations, since the velocity field in the fluid coincides (up to a multiplicative constant) with the electric field:

$$\mathbf{u} = m_{eo}\, \mathbf{E} \qquad \text{a.e. in } \Omega. \tag{4}$$

The purpose of this project is to develop a particle-steering method similar to that of the references, but considering that in the domain there are **obstacles** with a mobility **different to that of the walls**. In this situation (4) is no longer valid, and the effect of the actuators needs to be computed by solving the Stokes equations. The rest of the particle-steering method can be kept similar to that in the references, but it may require some adaptation.

The novelties are two: First, the change in the equations and boundary conditions with respect to the literature and, second, the presence of obstacles. In particular, the feedback law (Eq. 53 in the first article) is not immediately applicable, because it could lead to trajectories that collide with the obstacles.

Assuming that the top and bottom surfaces are parallel and at a distance h considered small with respect to the other dimensions, the problem can be approximated by the 2D problem

$$-\mu \nabla^2 \bar{\mathbf{u}} + \frac{12\,\mu}{h^2}(\bar{\mathbf{u}} - m_{tb}\mathbf{E}) + \nabla p = 0 \ , \qquad \nabla \cdot \bar{\mathbf{u}} = 0 \tag{5}$$

subject to $\mathbf{u}_\tau = m_o \mathbf{E}_\tau$ on the obstacles' surfaces.

The electric field is $\mathbf{E} = -\nabla\Phi$, where Φ is the electrostatic potential. The problem for Φ is

$$\nabla^2 \Phi = 0 \tag{6}$$

with Dirichlet boundary conditions at the electrodes and homogeneous Neumann boundary conditions at non-conducting surfaces.

We will consider as domain the square $(0, L)^2$, where the central part of length aL $(0 \leq a < 1)$ of each side is an electrode. We have thus 4 electrodes, East, North, West and South. The South electrode is set to zero potential without loss of generality. The remaining potentials, $\boldsymbol{\Phi} = (\Phi_E, \Phi_N, \Phi_W)^T$ are the actuators of the system. They generate a velocity field $\mathbf{u} = \mathcal{U}(\boldsymbol{\Phi})$ which is linear in $\boldsymbol{\Phi}$,

$$\mathbf{u}(\mathbf{x}) = A(\mathbf{x})\, \boldsymbol{\Phi} \ , \tag{7}$$

because the electro-osmotic flow equations are linear in $\boldsymbol{\Phi}$. The 2×3 matrix $A(\mathbf{x})$ is strongly nonlinear in $\mathbf{x}$, since each column is the 2D velocity vector at $\mathbf{x}$ resulting from setting one component of $\boldsymbol{\Phi}$ to 1 and the others to 0.

We consider a particle at an initial arbitrary position $\mathbf{z}$. The first task we want to accomplish is to steer that particle to the center $\mathbf{O}$ of the device. The position $\mathbf{X}(t)$ of the particle obeys

$$\frac{d\mathbf{X}}{dt} = \mathbf{u}(\mathbf{X}(t), t) + \mathbf{f}(t) \ , \qquad \mathbf{X}(0) = \mathbf{z} \ . \tag{8}$$

Above, $\mathbf{f}(t)$ is a perturbation force which can result from thermal noise, electrophoretic effects, swimming (if the particles are living organisms), etc. It is initially assumed to be zero.

The question (open-loop control) is: What are the necessary potentials $\boldsymbol{\Phi}(t)$ so that the particle moves from $\mathbf{z}$ to $\mathbf{O}$?

The second question (closed-loop control) is: Assume you can measure the particle's position $\mathbf{X}(t)$. Determine a feedback control of the form

$$\boldsymbol{\Phi}(t) = F(\mathbf{X}(t)) \tag{9}$$

so that $\mathbf{X}(t)$ moves towards $\mathbf{O}$ even if a (small) perturbation force $\mathbf{f}(t)$ is added to the system.