

Lecture 8 - Turbulence

Applied Computational Fluid Dynamics

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Turbulence

- What is turbulence?
- Effect of turbulence on Navier-Stokes equations.
- Reynolds averaging.
- Reynolds stresses.



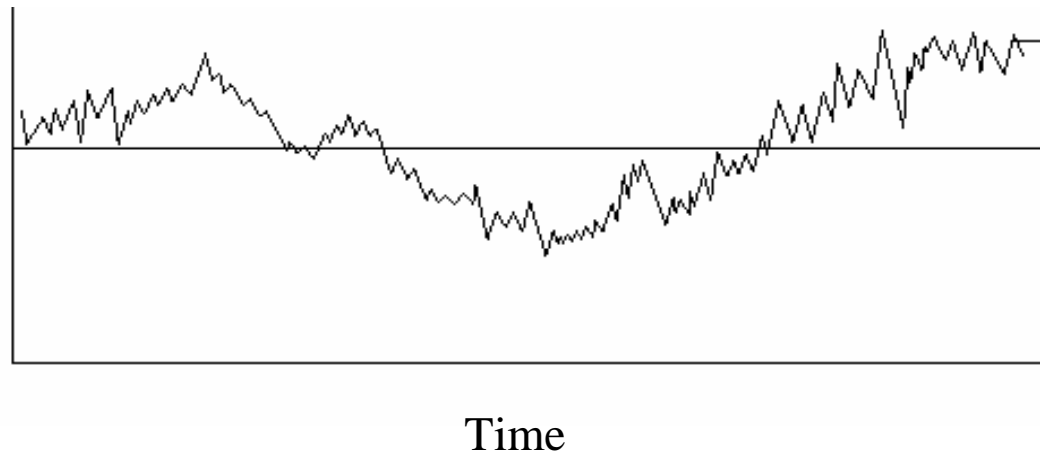
Instability

- All flows become unstable above a certain Reynolds number.
- At low Reynolds numbers flows are laminar.
- For high Reynolds numbers flows are turbulent.
- The transition occurs anywhere between 2000 and $1E6$, depending on the flow.
- For laminar flow problems, flows can be solved using the conservation equations developed previously.
- For turbulent flows, the computational effort involved in solving those for all time and length scales is prohibitive.
- An engineering approach to calculate time-averaged flow fields for turbulent flows will be developed.

What is turbulence?

- Unsteady, aperiodic motion in which all three velocity components fluctuate, mixing matter, momentum, and energy.
- Decompose velocity into mean and fluctuating parts:

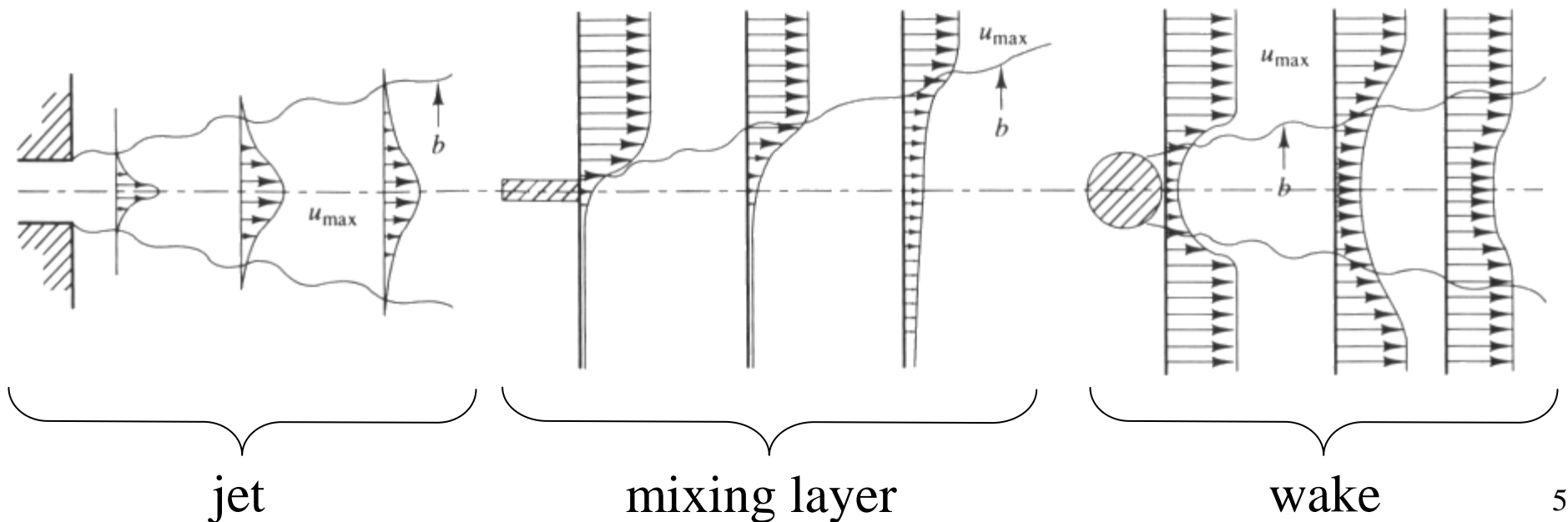
$$U_i(t) \equiv U_i + u_i(t).$$



- Similar fluctuations for pressure, temperature, and species concentration values.

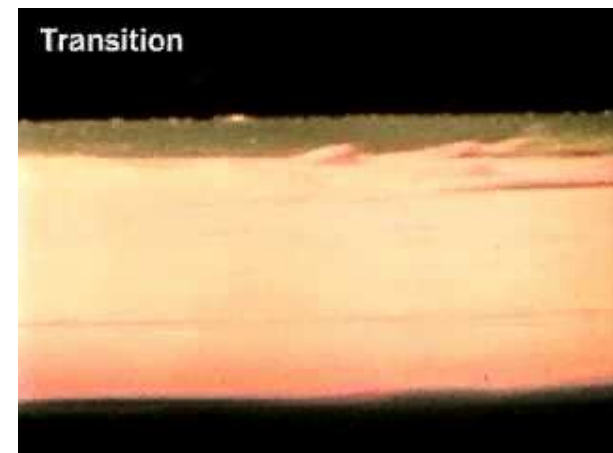
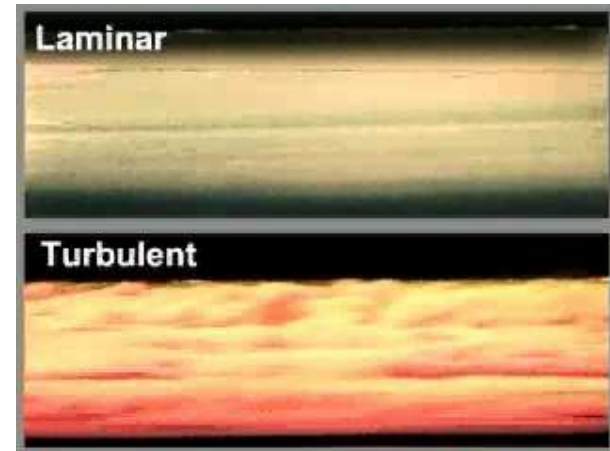
Examples of simple turbulent flows

- Some examples of simple turbulent flows are a jet entering a domain with stagnant fluid, a mixing layer, and the wake behind objects such as cylinders.
- Such flows are often used as test cases to validate the ability of computational fluid dynamics software to accurately predict fluid flows.

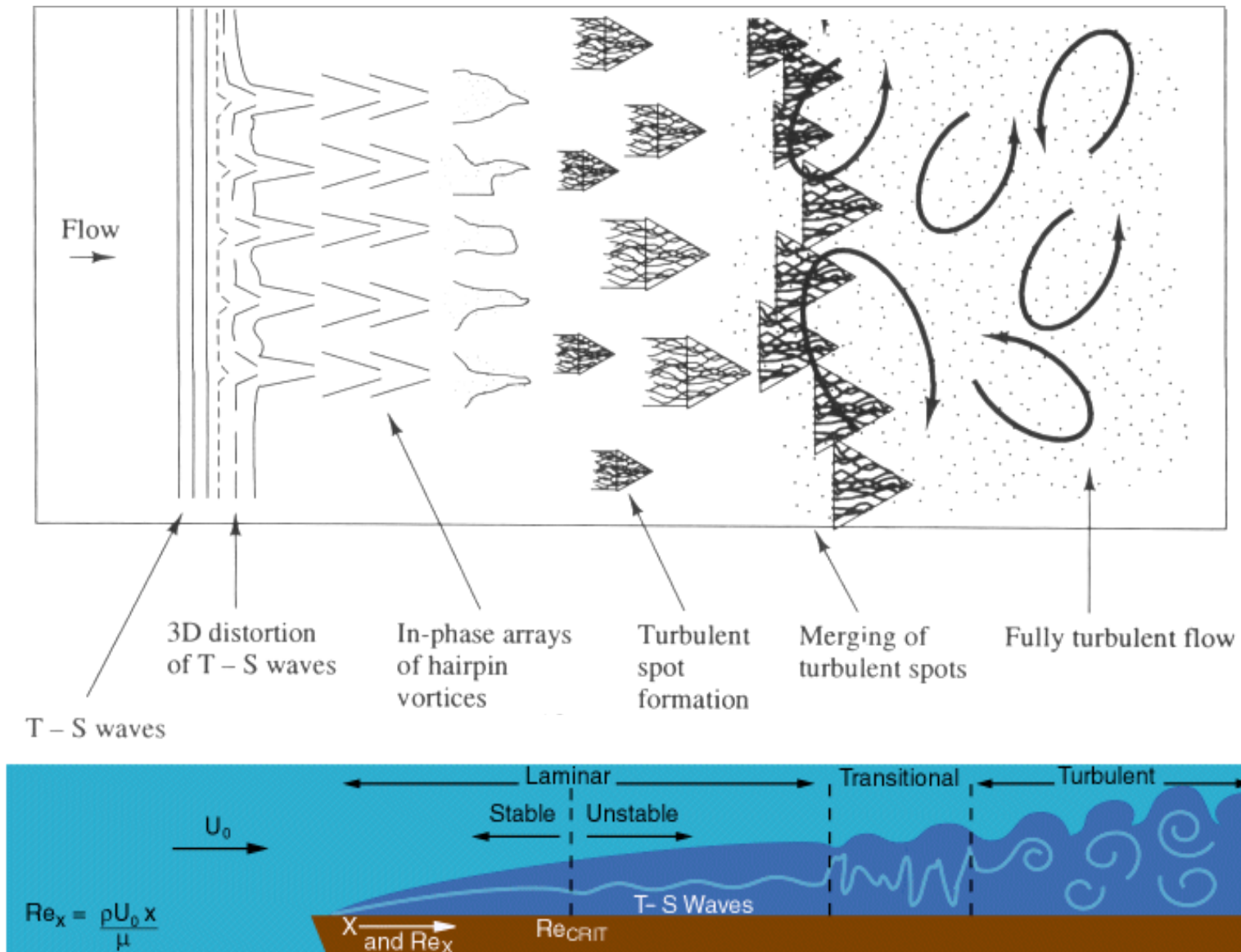


Transition

- The photographs show the flow in a boundary layer.
- Below Re_{crit} the flow is laminar and adjacent fluid layers slide past each other in an orderly fashion.
- The flow is stable. Viscous effects lead to small disturbances being dissipated.
- Above the transition point Re_{crit} small disturbances in the flow start to grow.
- A complicated series of events takes place that eventually leads to the flow becoming fully turbulent.



Transition in boundary layer flow over flat plate



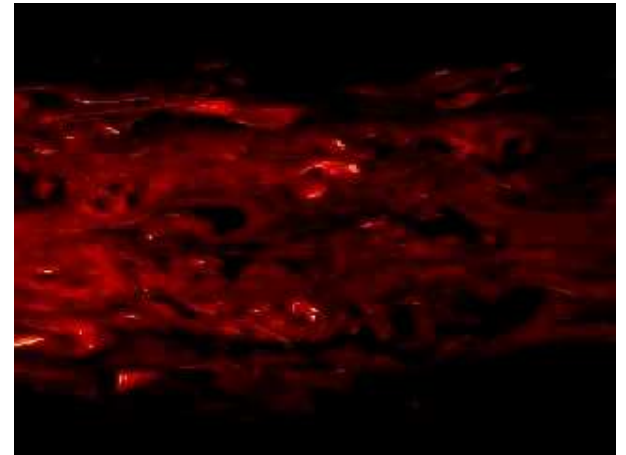
Transition in boundary layer flow over flat plate



T-S waves

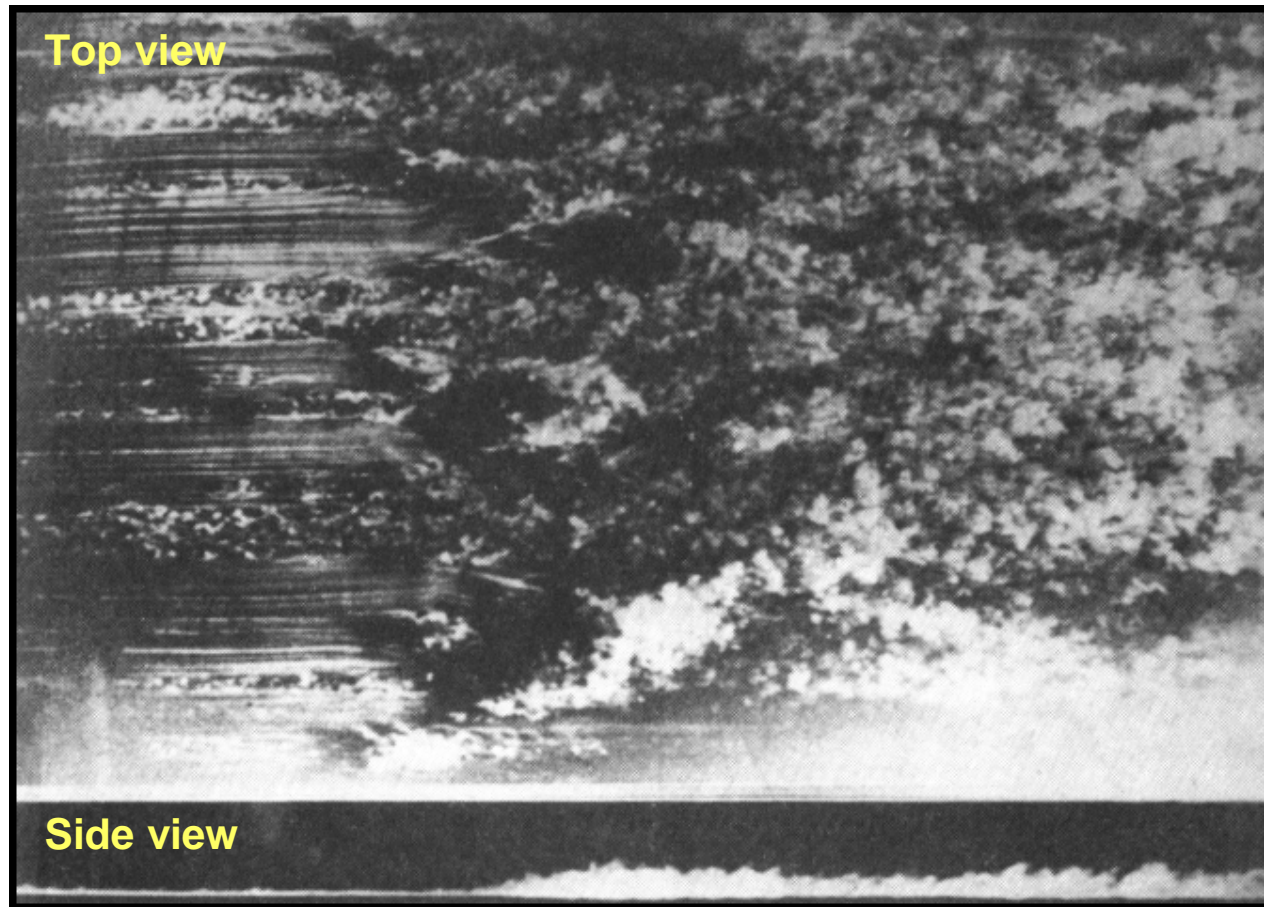


Turbulent spots



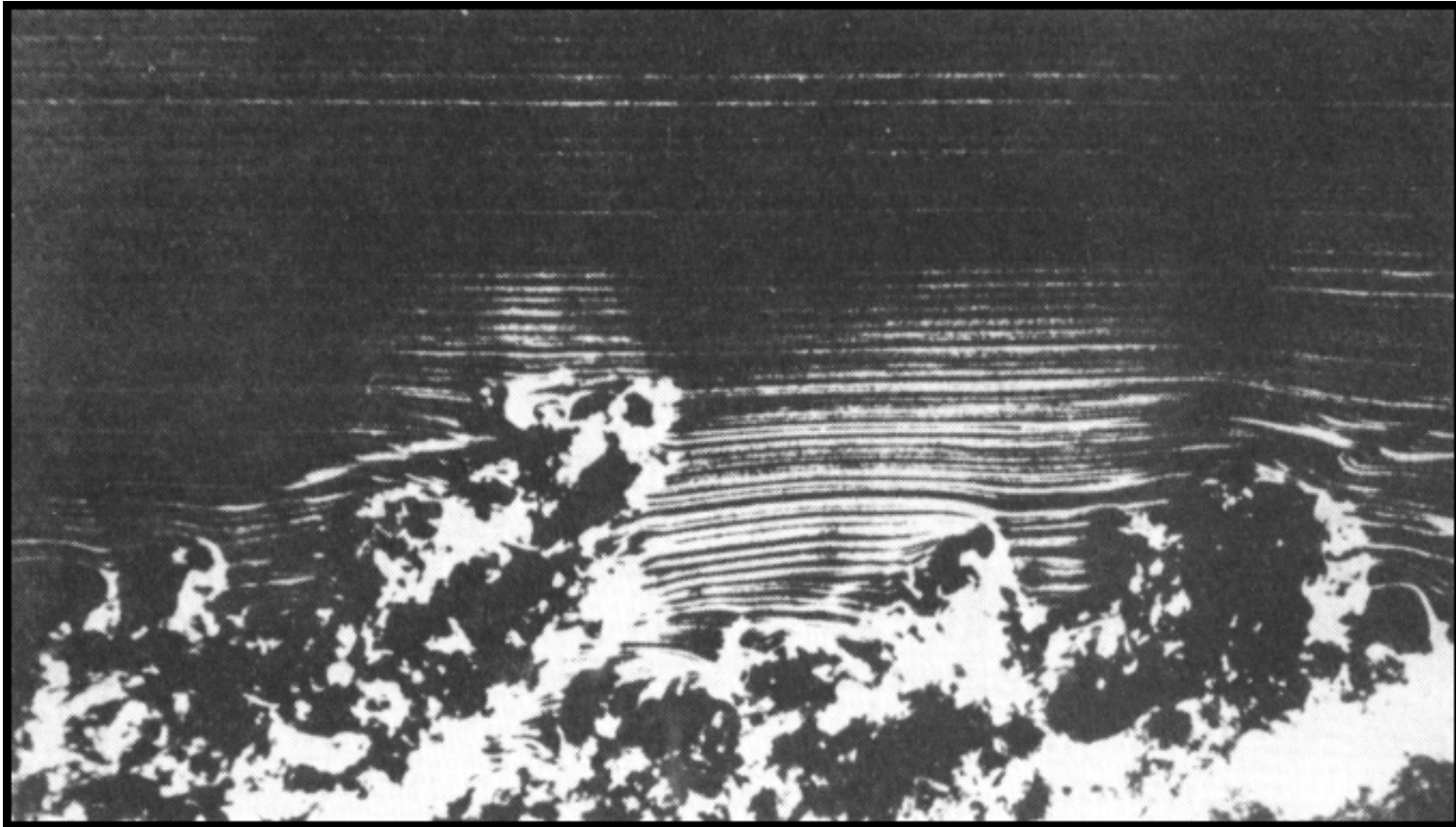
Fully turbulent flow

Turbulent boundary layer



**Merging of turbulent spots and transition to turbulence
in a natural flat plate boundary layer.**

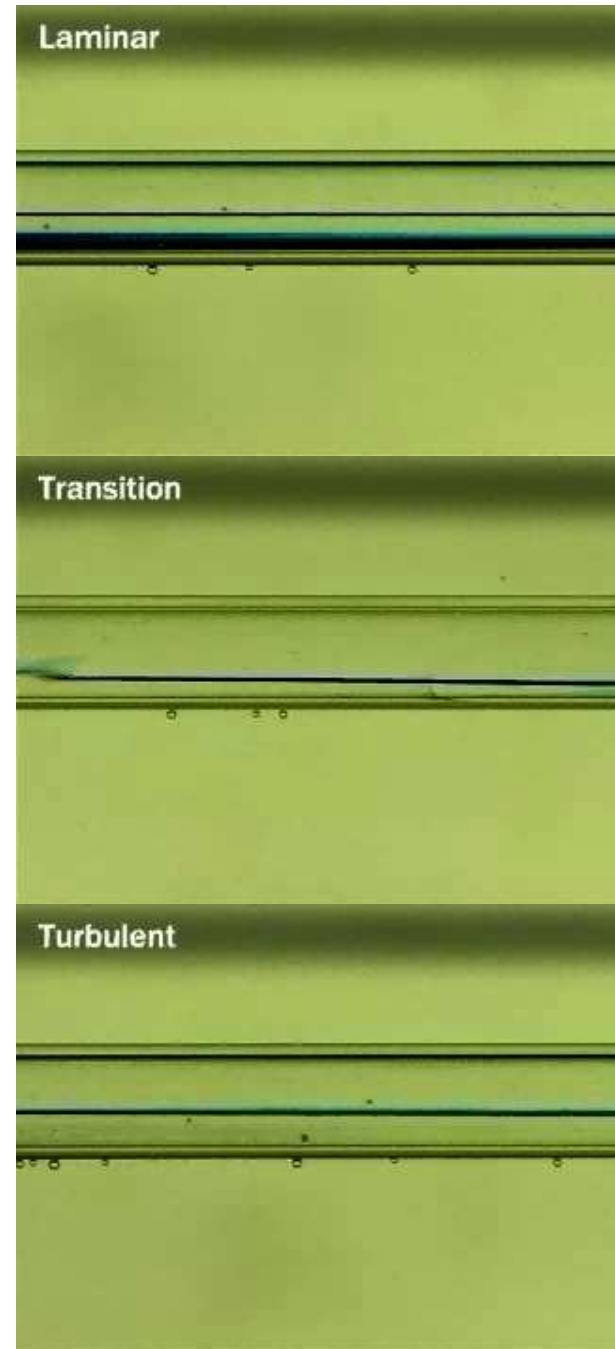
Turbulent boundary layer



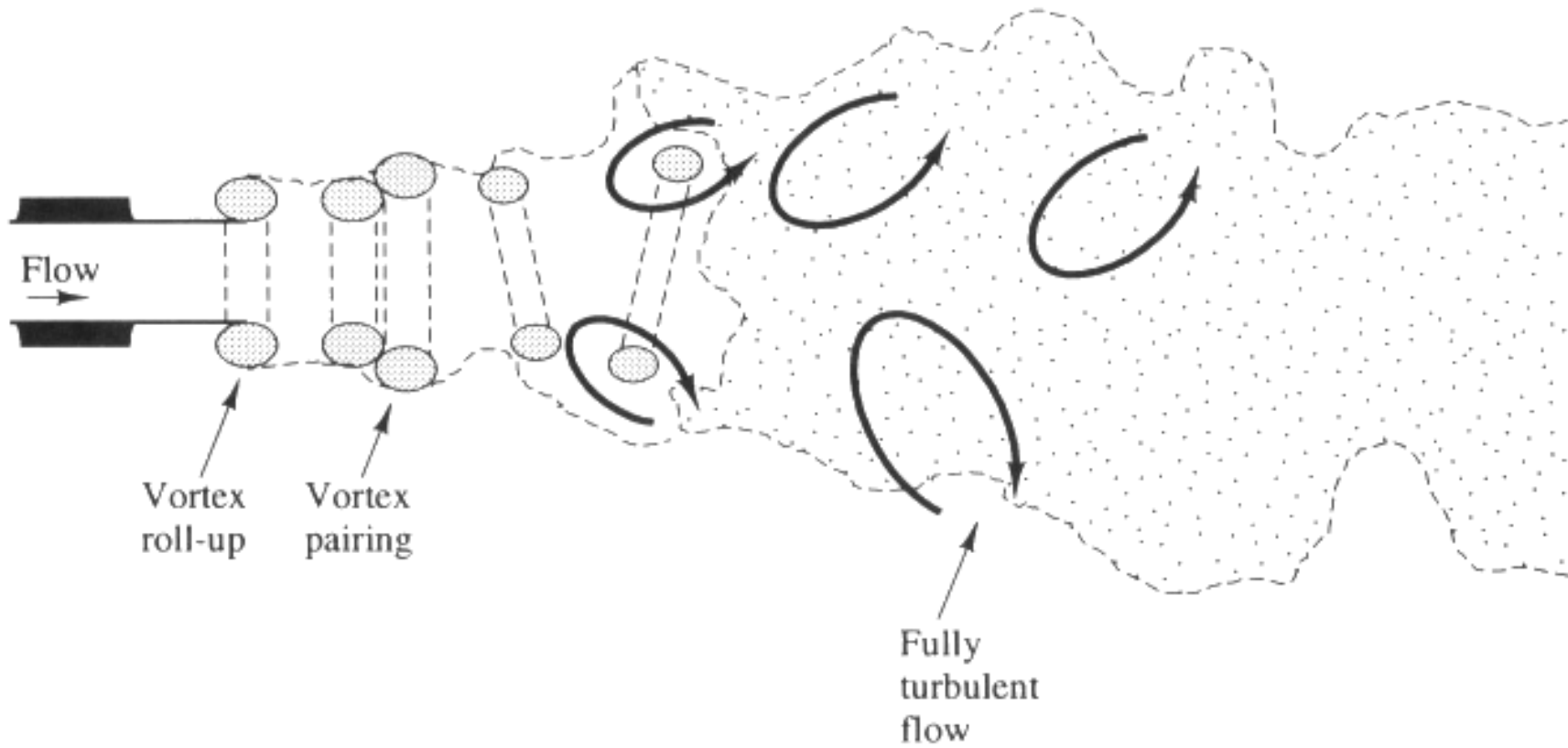
Close-up view of the turbulent boundary layer.

Transition in a channel flow

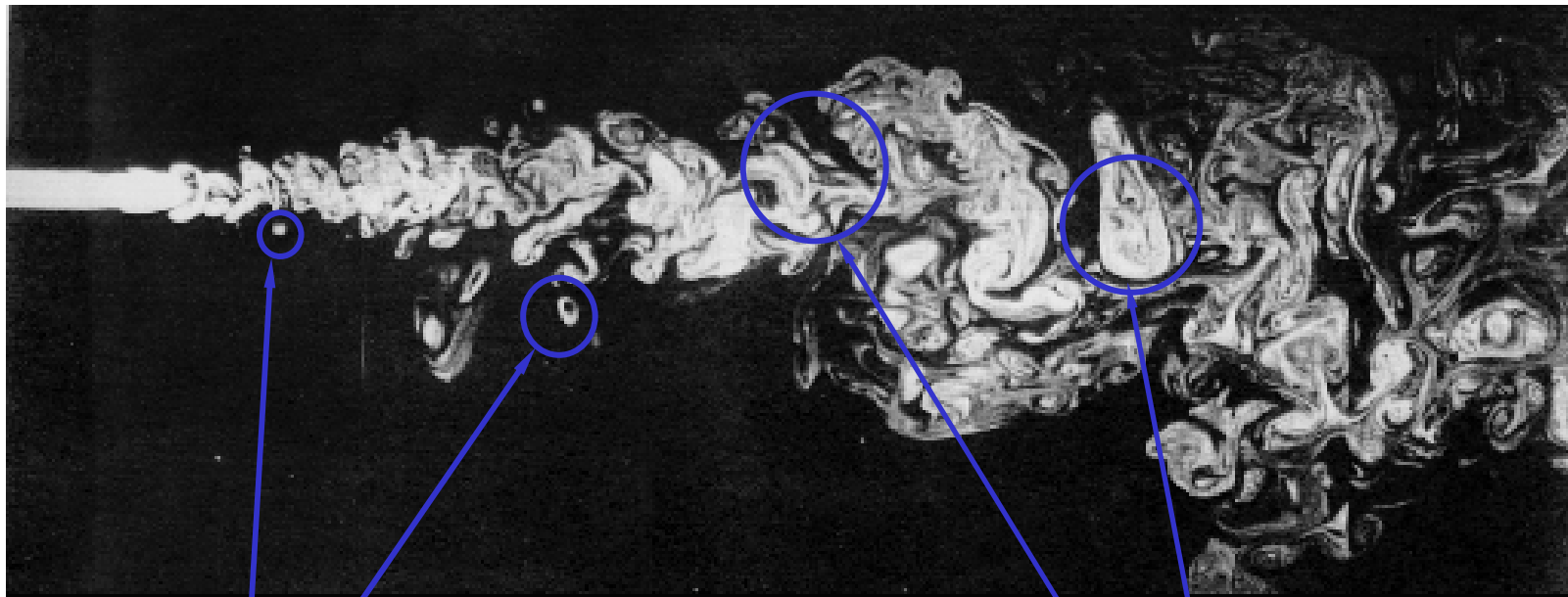
- Instability and turbulence is also seen in internal flows such as channels and ducts.
- The Reynolds number is constant throughout the pipe and is a function of flow rate, fluid properties and diameter.
- Three flow regimes are shown:
 - $Re < 2200$ with laminar flow.
 - $Re = 2200$ with a flow that alternates between turbulent and laminar. This is called transitional flow.
 - $Re > 2200$ with fully turbulent flow.



Transition in a jet flow



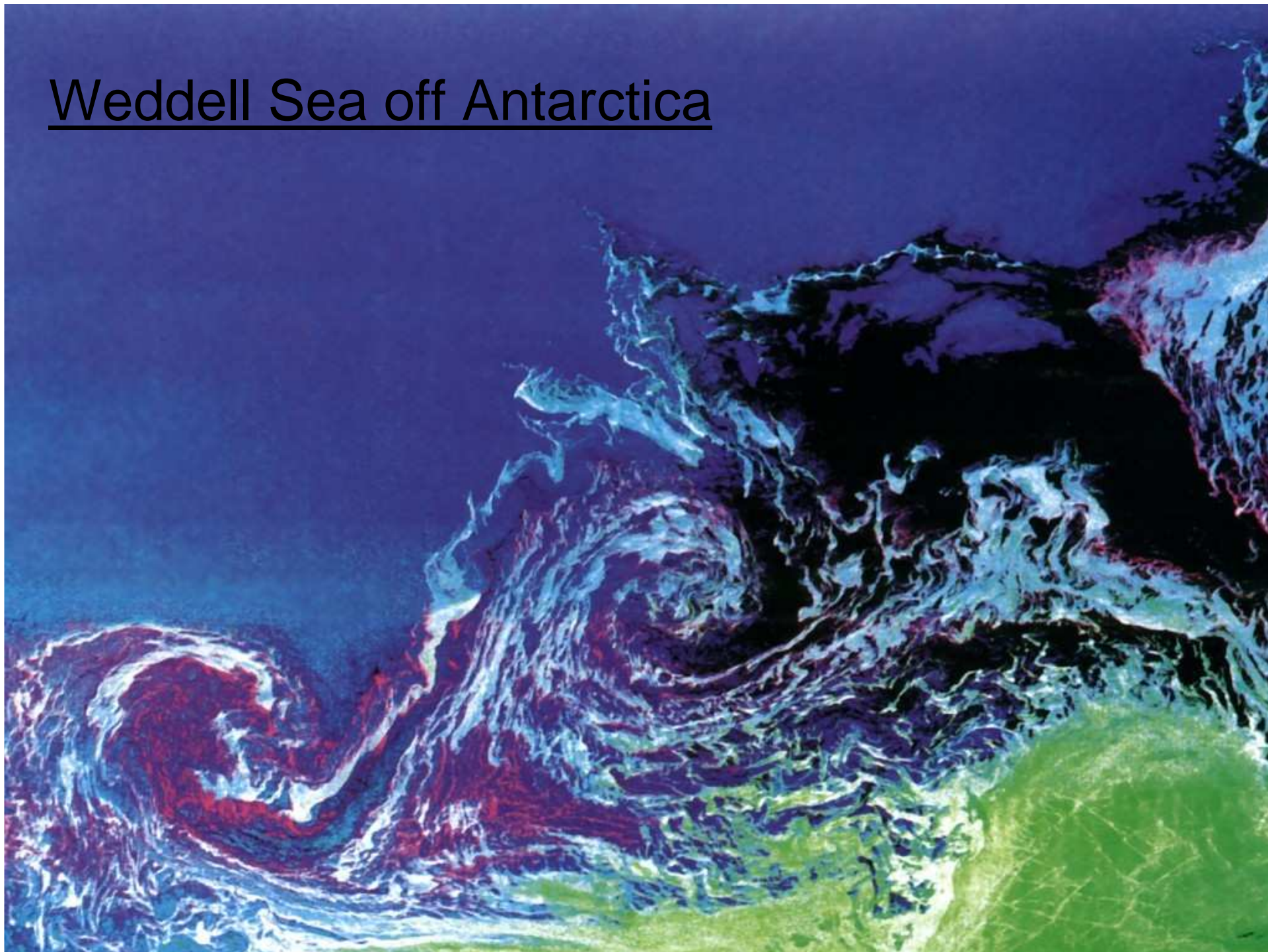
Large-scale vs. small-scale structure



Small Structure

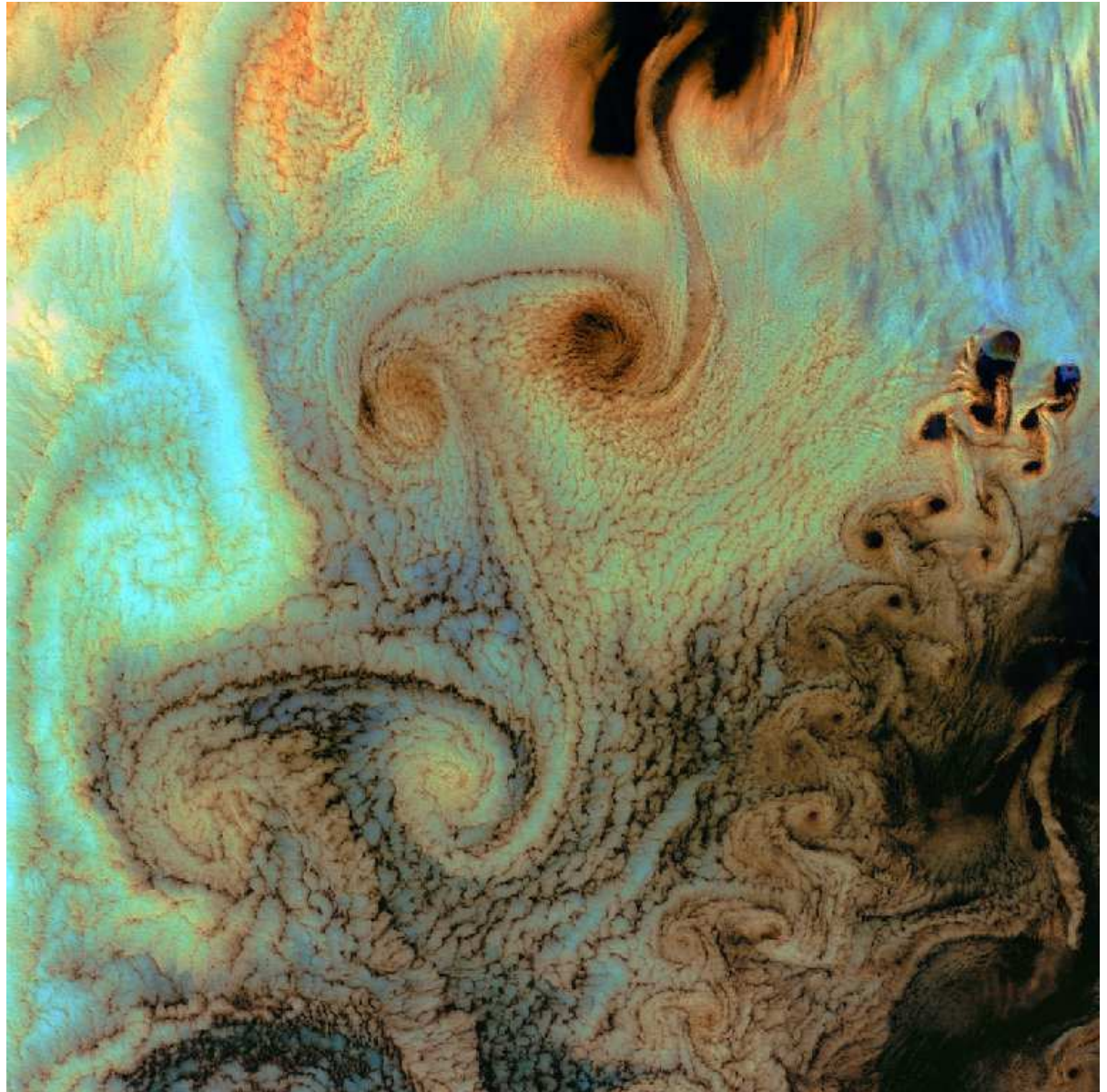
Large Structure

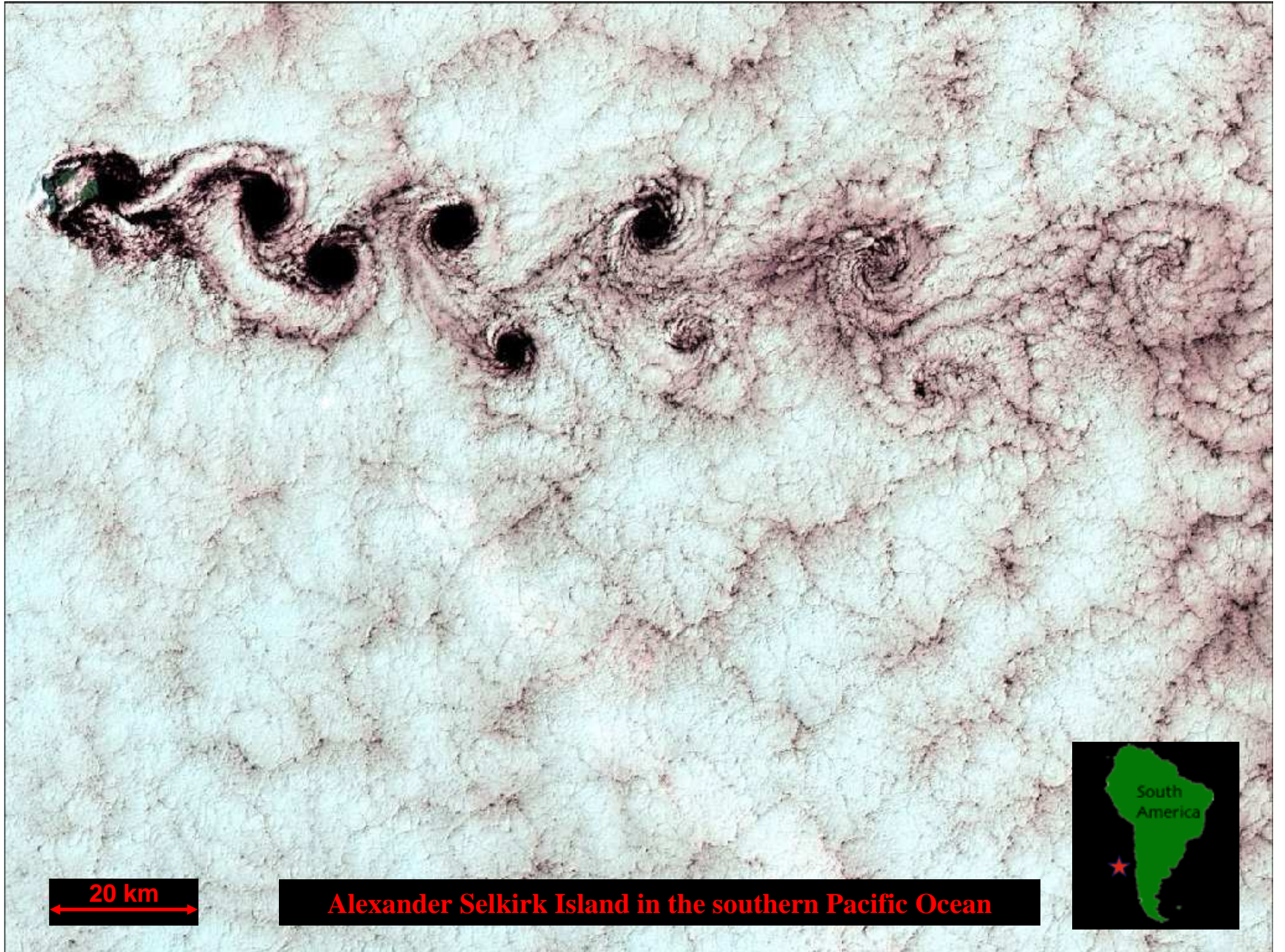
Weddell Sea off Antarctica



Alaska's Aleutian Islands

- As air flows over and around objects in its path, spiraling eddies, known as Von Karman vortices, may form.
- The vortices in this image were created when prevailing winds sweeping east across the northern Pacific Ocean encountered Alaska's Aleutian Islands





20 km

Alexander Selkirk Island in the southern Pacific Ocean

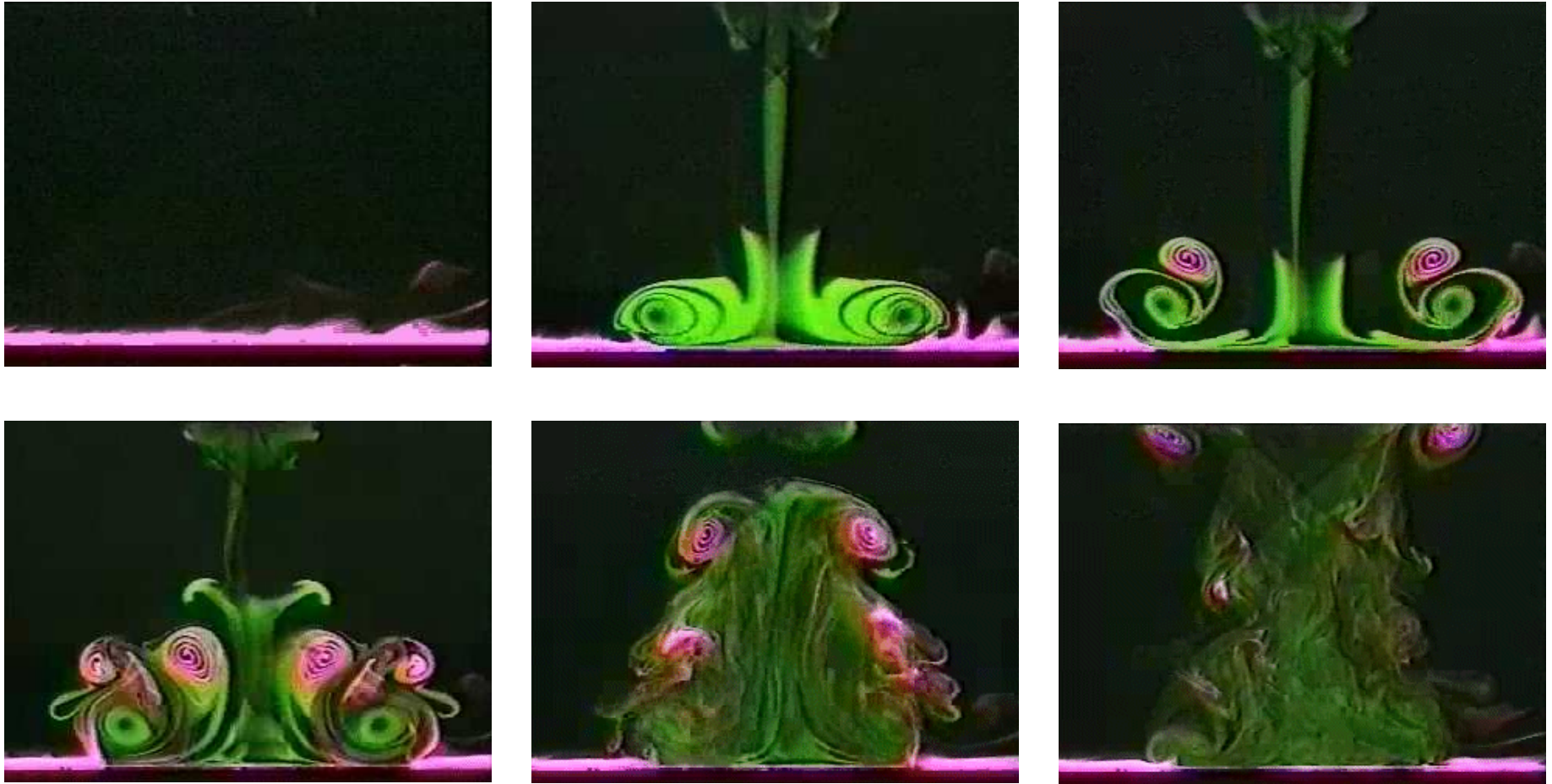




← 4.4 miles →

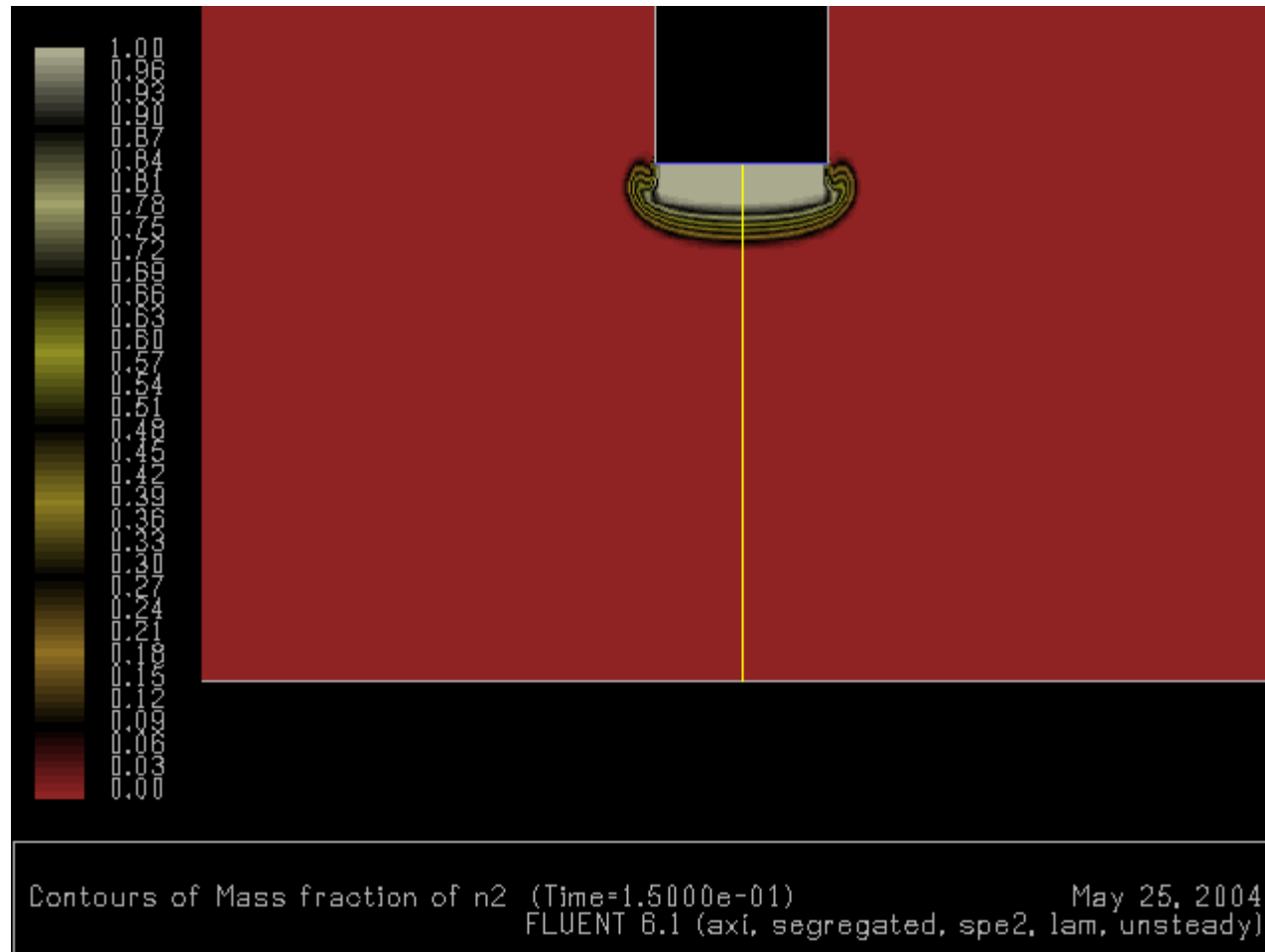


Smoke ring



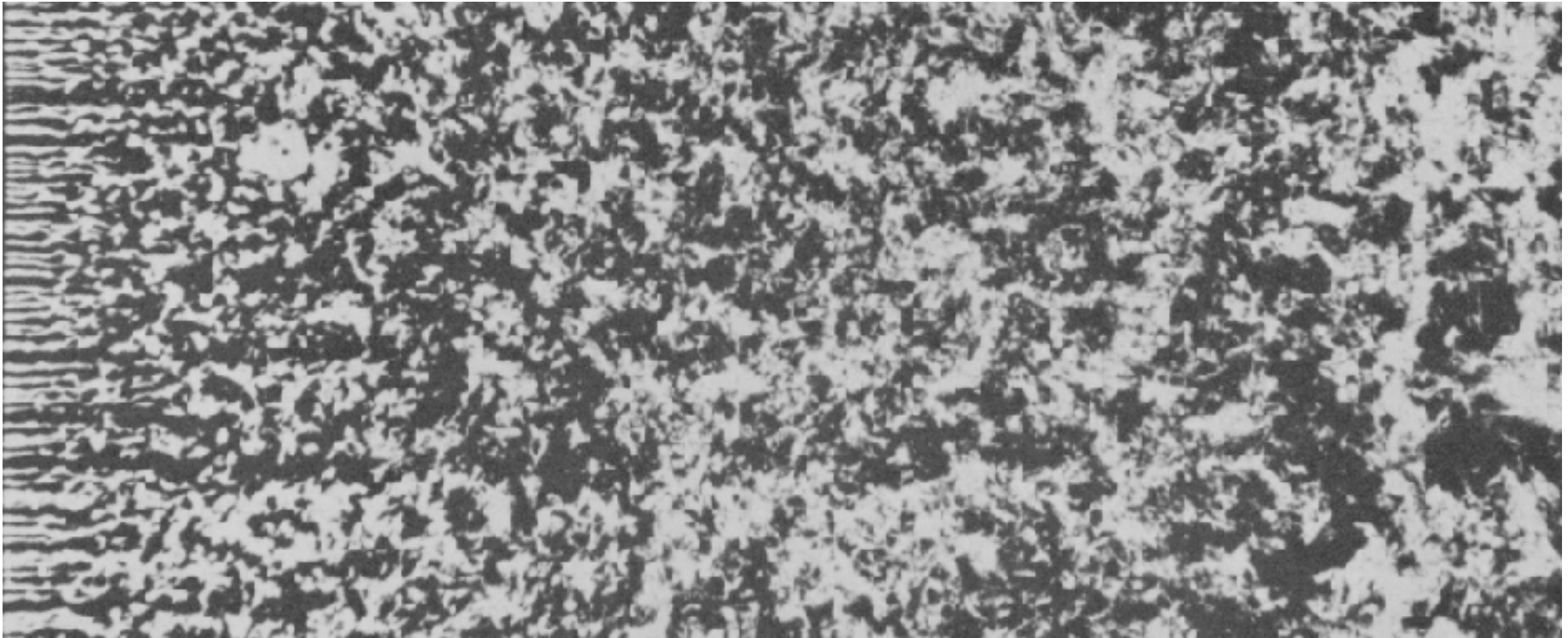
A smoke ring (green) impinges on a plate where it interacts with the slow moving smoke in the boundary layer (pink). The vortex ring stretches and new rings form. The size of the vortex structures decreases over time.

Simulation – species mixing



Arthur Shaw and Robert Haehnel, 2004

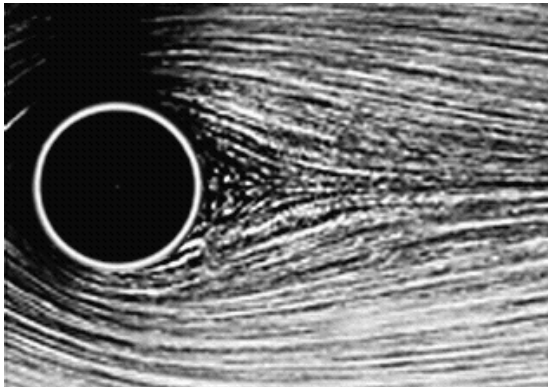
Homogeneous, decaying, grid-generated turbulence



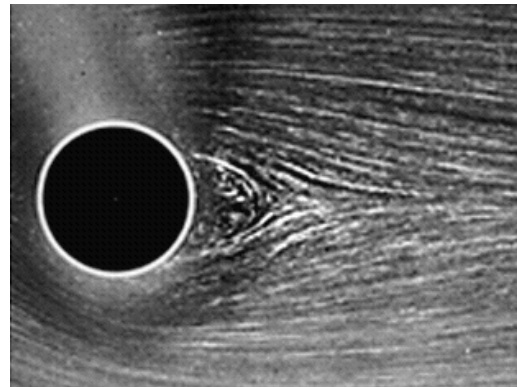
Turbulence is generated at the grid as a result of high stresses in the immediate vicinity of the grid. The turbulence is made visible by injecting smoke into the flow at the grid. The eddies are visible because they contain the smoke. Beyond this point, there is no source of turbulence as the flow is uniform. The flow is dominated by convection and dissipation. For homogeneous decaying turbulence, the turbulent kinetic energy decreases with distance from grid as x^{-1} and the turbulent eddies grows in size as $x^{1/2}$.

Flow transitions around a cylinder

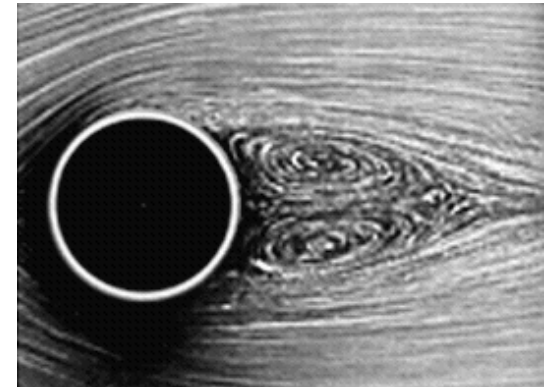
- For flow around a cylinder, the flow starts separating at $Re = 5$. For Re below 30, the flow is stable. Oscillations appear for higher Re .
- The separation point moves upstream, increasing drag up to $Re = 2000$.



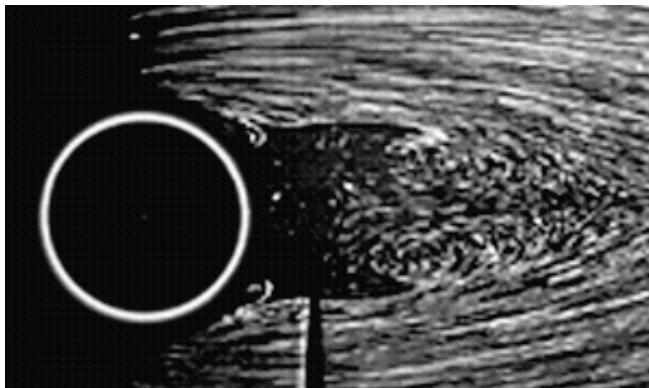
Re = 9.6



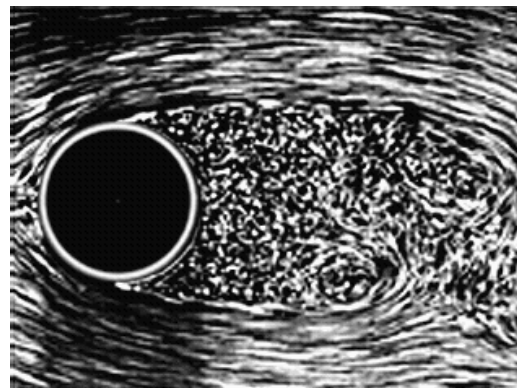
Re = 13.1



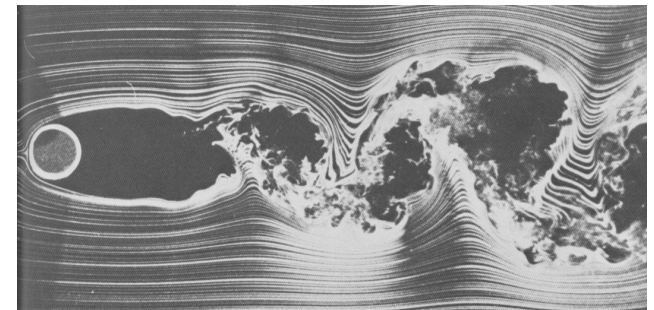
Re = 26



Re = 30.2



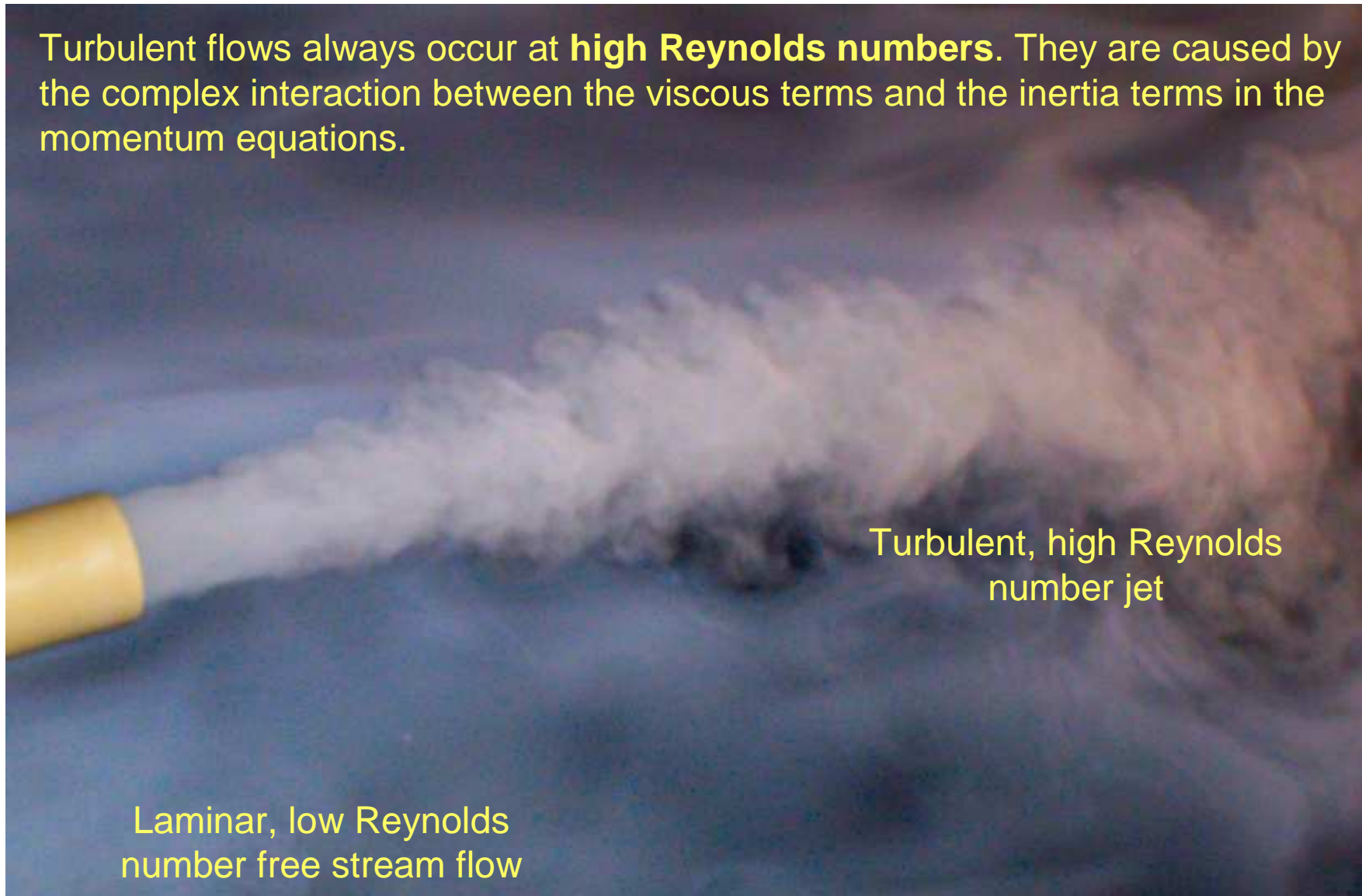
Re = 2000



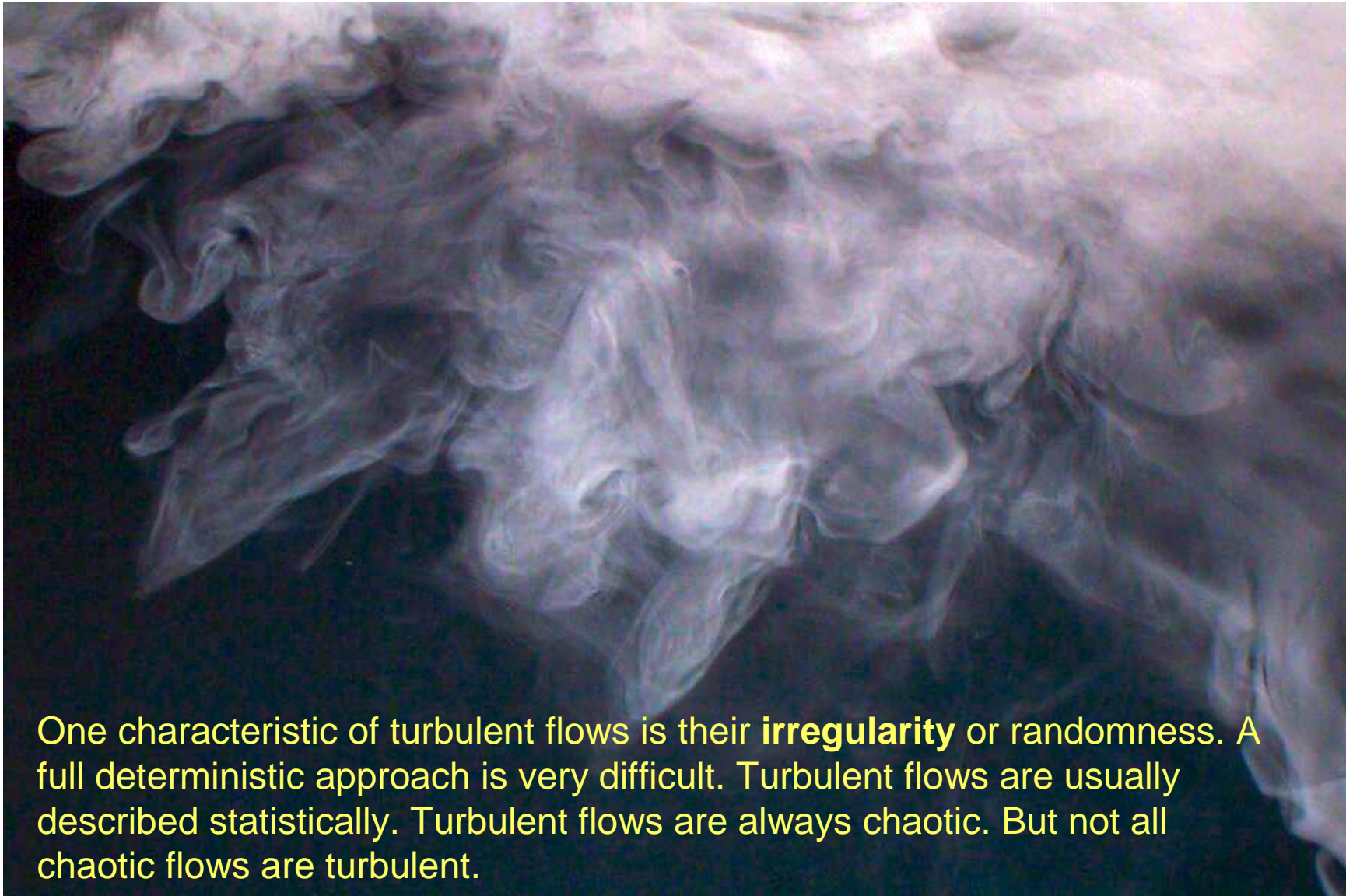
Re = 10,000

Turbulence: high Reynolds numbers

Turbulent flows always occur at **high Reynolds numbers**. They are caused by the complex interaction between the viscous terms and the inertia terms in the momentum equations.

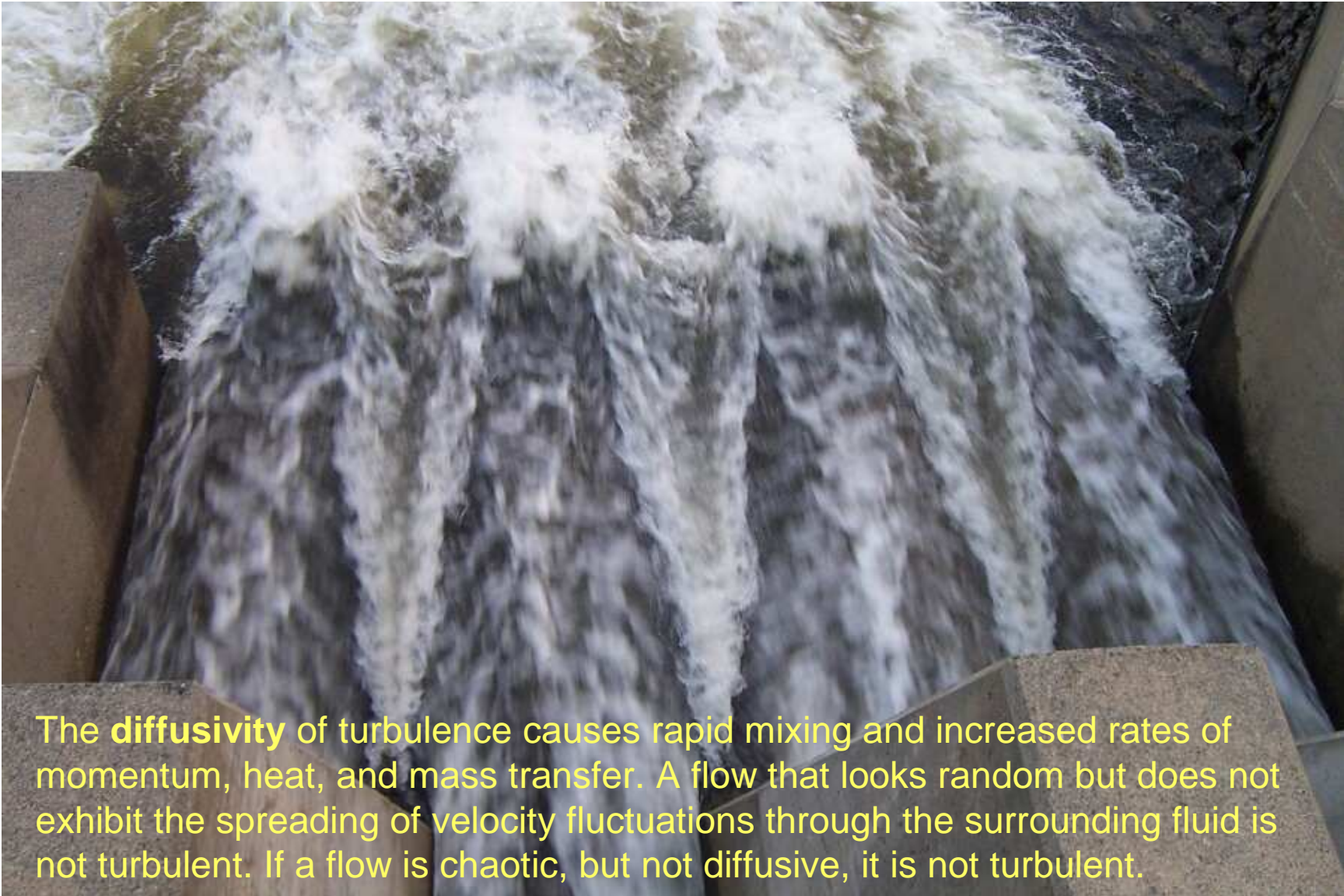


Turbulent flows are chaotic



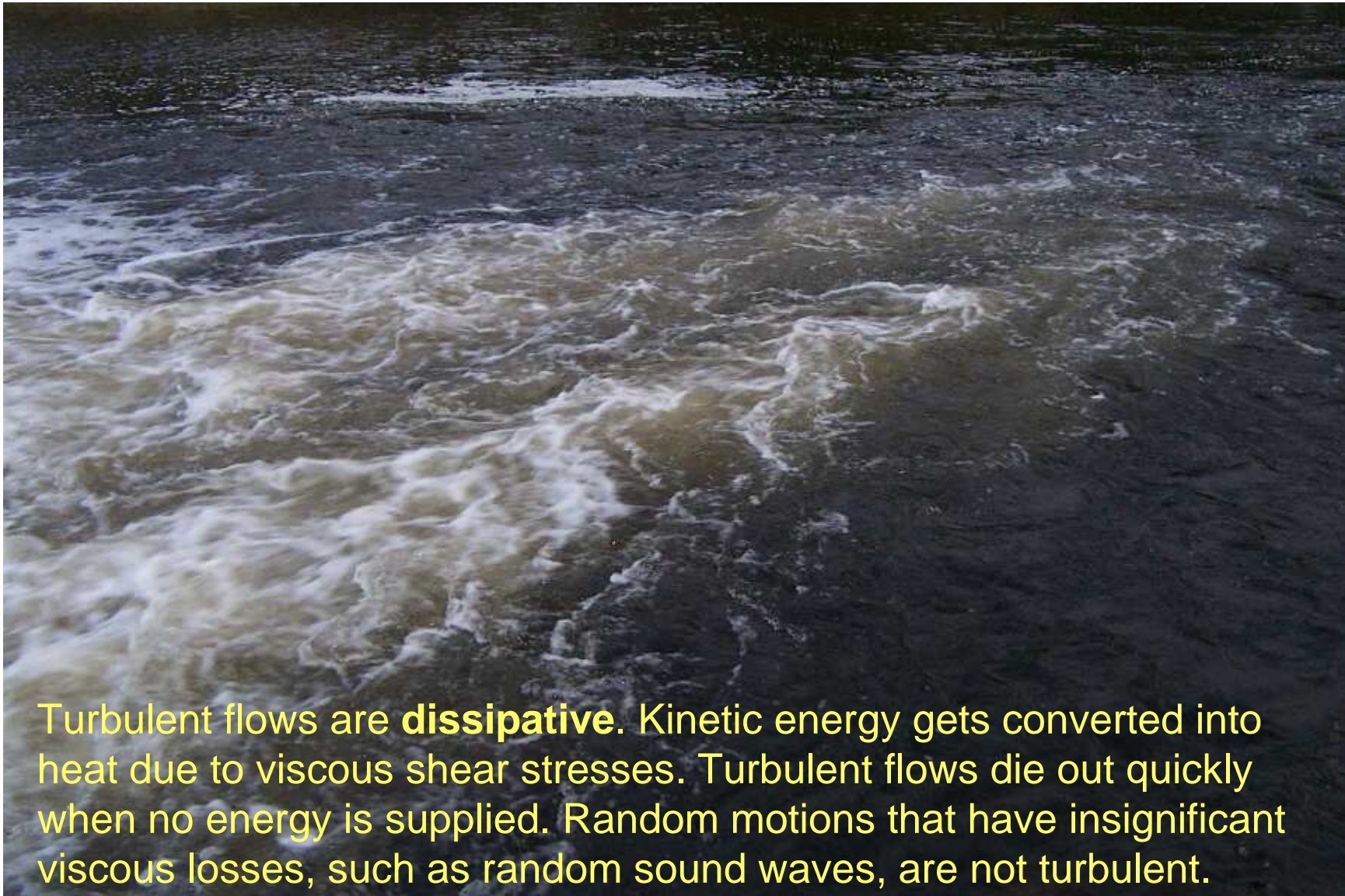
One characteristic of turbulent flows is their **irregularity** or randomness. A full deterministic approach is very difficult. Turbulent flows are usually described statistically. Turbulent flows are always chaotic. But not all chaotic flows are turbulent.

Turbulence: diffusivity



The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent. If a flow is chaotic, but not diffusive, it is not turbulent.

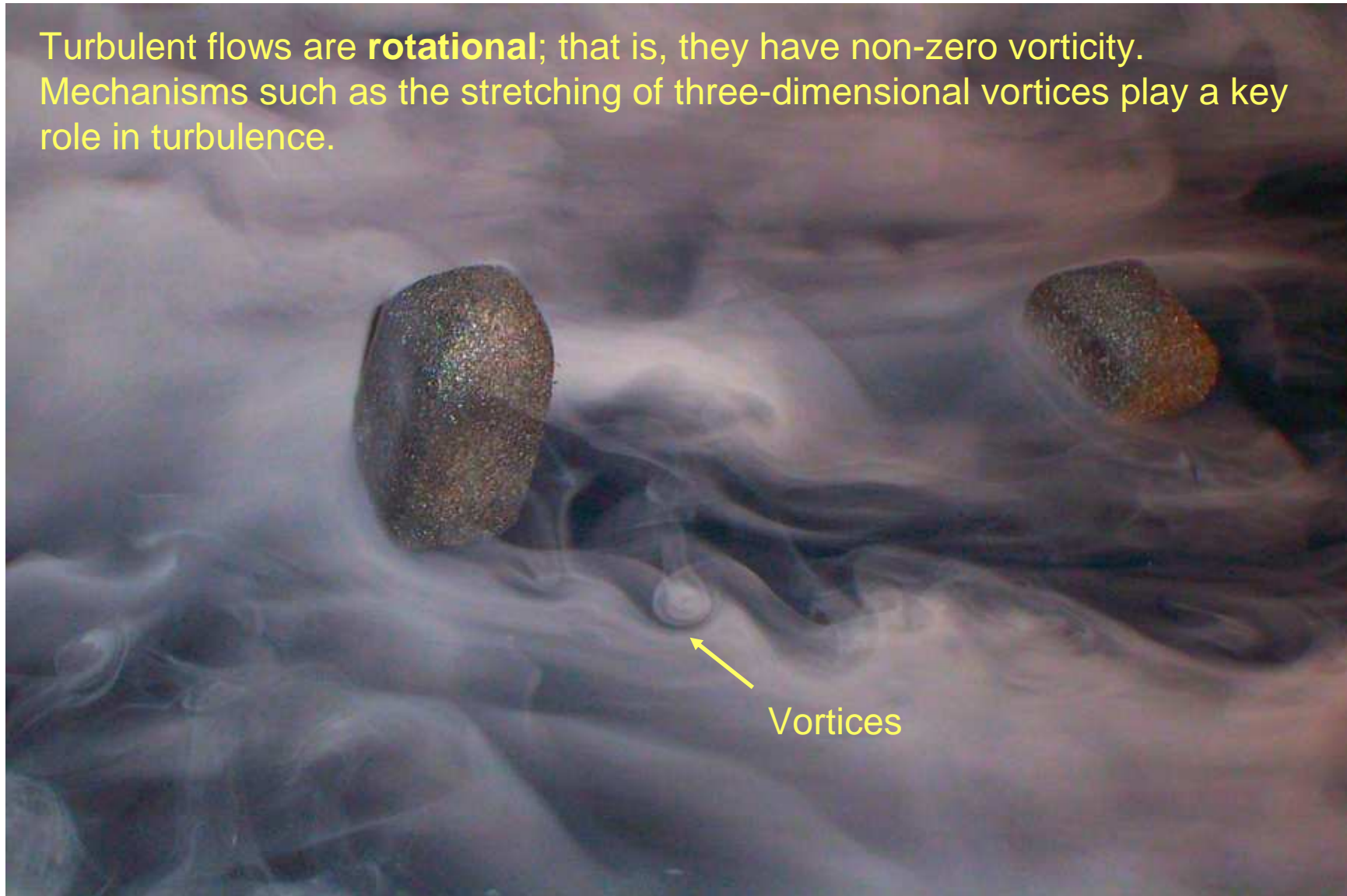
Turbulence: dissipation



Turbulent flows are **dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied. Random motions that have insignificant viscous losses, such as random sound waves, are not turbulent.

Turbulence: rotation and vorticity

Turbulent flows are **rotational**; that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.



What is turbulence?

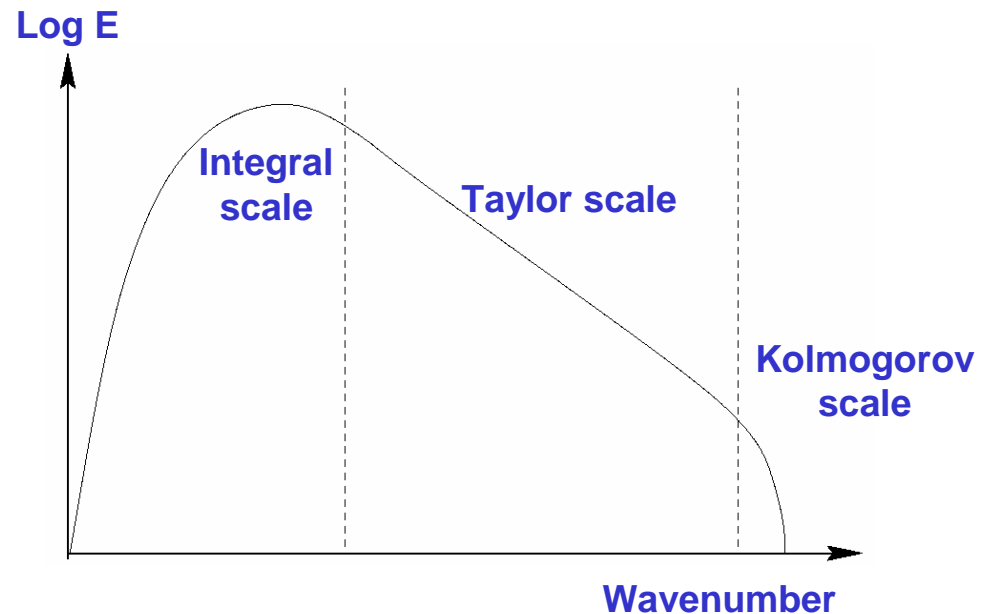
- Turbulent flows have the following characteristics:
 - One characteristic of turbulent flows is their **irregularity** or randomness. A full deterministic approach is very difficult. Turbulent flows are usually described statistically. Turbulent flows are always **chaotic**. But not all chaotic flows are turbulent. Waves in the ocean, for example, can be chaotic but are not necessarily turbulent.
 - The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent. If a flow is chaotic, but not diffusive, it is not turbulent. The trail left behind a jet plane that seems chaotic, but does not diffuse for miles is then not turbulent.
 - Turbulent flows always occur at **high Reynolds numbers**. They are caused by the complex interaction between the viscous terms and the inertia terms in the momentum equations.
 - Turbulent flows are **rotational**; that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.

What is turbulence? - Continued

- Turbulent flows are **dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied. Random motions that have insignificant viscous losses, such as random sound waves, are not turbulent.
- Turbulence is a **continuum** phenomenon. Even the smallest eddies are significantly larger than the molecular scales. Turbulence is therefore governed by the equations of fluid mechanics.
- Turbulent flows are flows. Turbulence is a **feature of fluid flow**, not of the fluid. When the Reynolds number is high enough, most of the dynamics of turbulence are the same whether the fluid is an actual fluid or a gas. Most of the dynamics are then independent of the properties of the fluid.

Kolmogorov energy spectrum

- Energy cascade, from large scale to small scale.
- E is energy contained in eddies of wavelength λ .
- Length scales:
 - Largest eddies. Integral length scale ($k^{3/2}/\varepsilon$).
 - Length scales at which turbulence is isotropic. Taylor microscale $(15\nu u'^2/\varepsilon)^{1/2}$.
 - Smallest eddies. Kolmogorov length scale $(\nu^3/\varepsilon)^{1/4}$. These eddies have a velocity scale $(\nu \cdot \varepsilon)^{1/4}$ and a time scale $(\nu/\varepsilon)^{1/2}$.

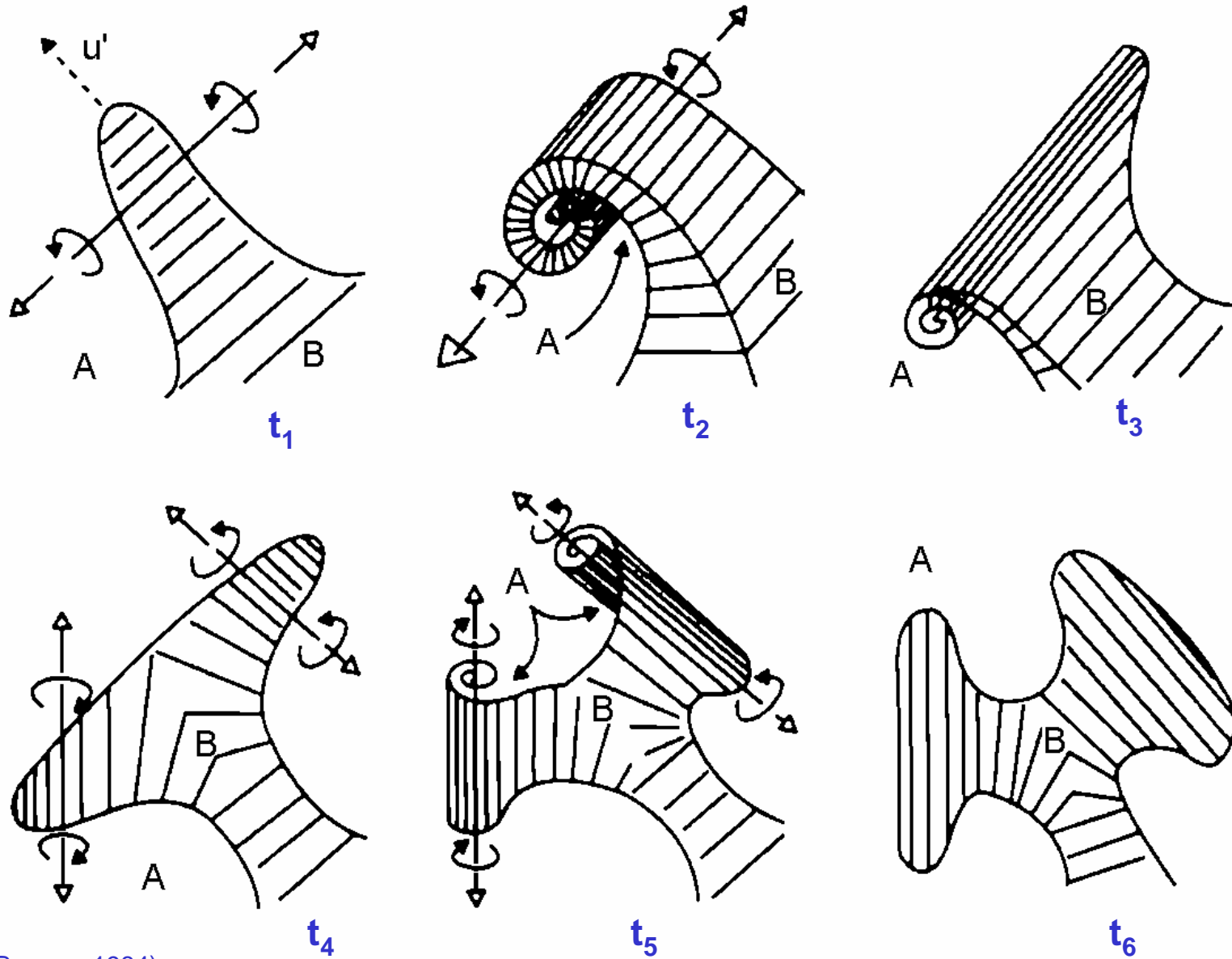


ε is the energy dissipation rate (m^2/s^3)
 k is the turbulent kinetic energy (m^2/s^2)
 ν is the kinematic viscosity (m^2/s)

Vorticity and vortex stretching

- Existence of eddies implies rotation or vorticity.
- Vorticity concentrated along contorted vortex lines or bundles.
- As end points of a vortex line move randomly further apart the vortex line increases in length but decreases in diameter. Vorticity increases because angular momentum is nearly conserved. Kinetic energy increases at rate equivalent to the work done by large-scale motion that stretches the bundle.
- Viscous dissipation in the smallest eddies converts kinetic energy into thermal energy.
- Vortex-stretching cascade process maintains the turbulence and dissipation is approximately equal to the rate of production of turbulent kinetic energy.
- Typically energy gets transferred from the large eddies to the smaller eddies. However, sometimes smaller eddies can interact with each other and transfer energy to the (i.e. form) larger eddies, a process known as backscatter.

Vortex stretching



(Baldyga and Bourne, 1984)

Is the flow turbulent?

External flows:

$$Re_x \geq 5 \times 10^5 \text{ along a surface}$$

$$Re_D \geq 20,000 \text{ around an obstacle}$$

Internal flows:

$$Re_{D_h} \geq \square 2,200$$

Natural convection:

$$Ra \geq 10^8 - 10^{10}$$

where $Re_L \equiv \frac{\rho UL}{\mu}$

$$L = x, D, D_h, \text{ etc.}$$

Other factors such as free-stream turbulence, surface conditions, and disturbances may cause earlier transition to turbulent flow.

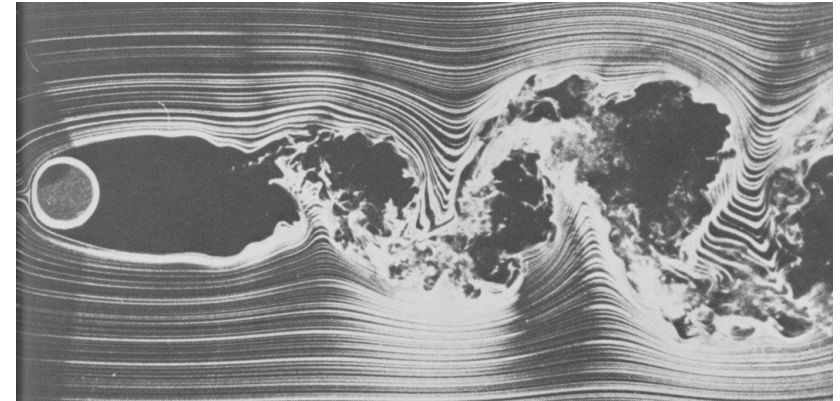
where $Ra \equiv \frac{g\beta\Delta TL^3\rho}{\mu\alpha}$

Turbulence modeling objective

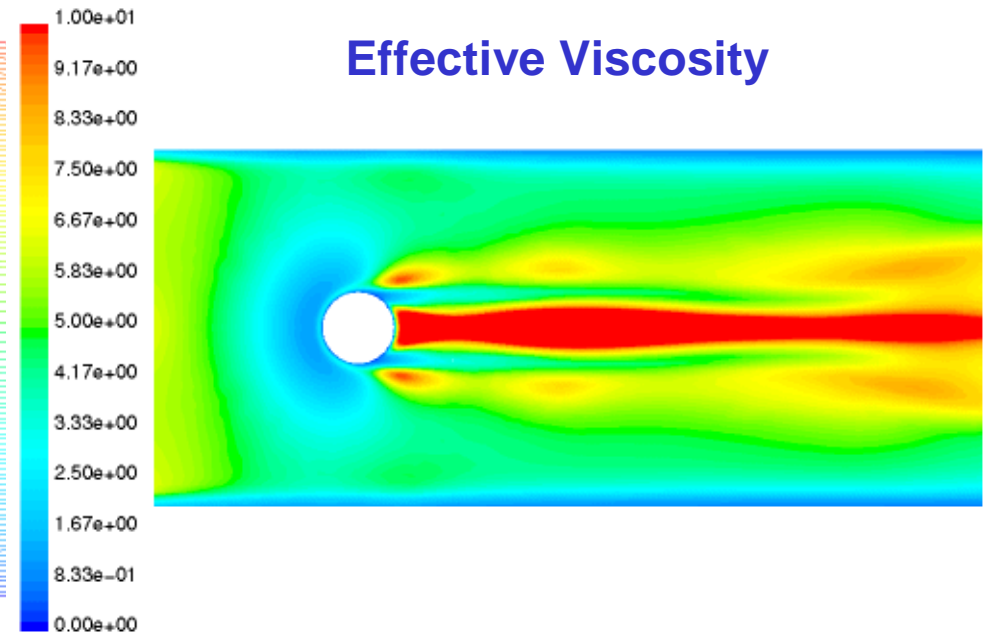
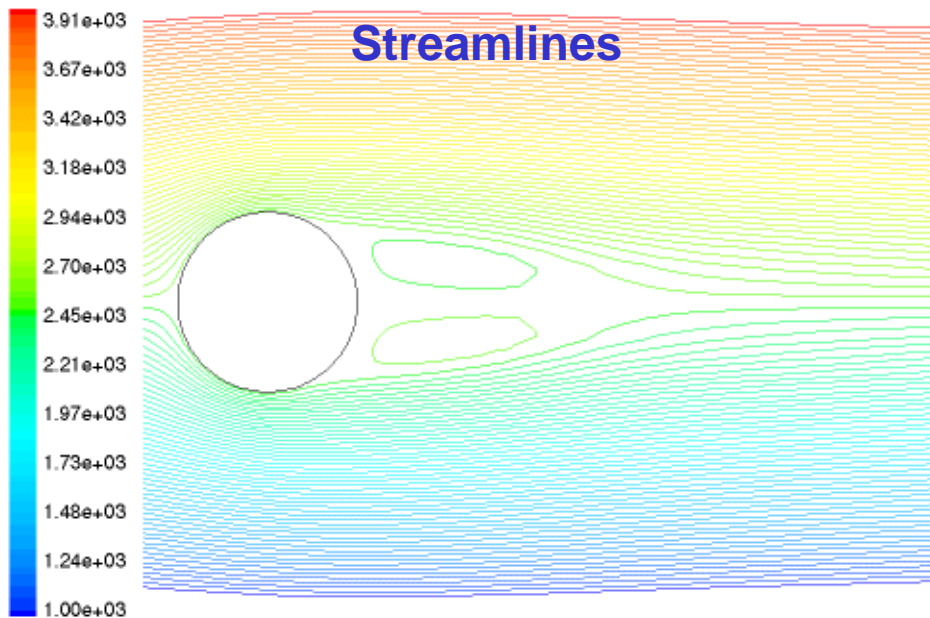
- The objective of turbulence modeling is to develop equations that will predict the time averaged velocity, pressure, and temperature fields without calculating the complete turbulent flow pattern as a function of time.
 - This saves us a lot of work!
 - Most of the time it is all we need to know.
 - We may also calculate other statistical properties, such as RMS values.
- Important to understand: the time averaged flow pattern is a statistical property of the flow.
 - It is not an existing flow pattern!
 - It does not usually satisfy the steady Navier-Stokes equations!
 - The flow never actually looks that way!!

Example: flow around a cylinder at $Re=1E4$

- The figures show:
 - An experimental snapshot.
 - Streamlines for time averaged flow field. Note the difference between the time averaged and the instantaneous flow field.
 - Effective viscosity used to predict time averaged flow field.



Experimental Snapshot



Decomposition

Flow property φ . The mean Φ is defined as :

$$\Phi = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi(t) dt$$

Δt should be larger than the time scale of the slowest turbulent fluctuations.

Time dependence : $\varphi(t) = \Phi + \varphi'(t)$

Write shorthand as : $\varphi = \Phi + \varphi'$

$$\bar{\varphi}' = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi'(t) dt = 0 \quad \text{by definition}$$

Information regarding the fluctuating part of the flow can be obtained from the root – mean - square (rms) of the fluctuations :

$$\varphi_{rms} = \sqrt{\overline{(\varphi')^2}} = \left[\frac{1}{\Delta t} \int_0^{\Delta t} (\varphi')^2 dt \right]^{1/2}$$

Velocity decomposition

- Velocity and pressure decomposition:

$$\text{Velocity : } \mathbf{u} = \mathbf{U} + \mathbf{u}'$$

$$\text{Pressure : } p = P + p'$$

- Turbulent kinetic energy k (per unit mass) is defined as:

$$k = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

$$\text{Turbulence intensity : } T_i = \frac{(\frac{2}{3}k)^{1/2}}{U_{ref}}$$

- Continuity equation:

$$\text{div } \mathbf{u} = 0; \text{ Time average : } \overline{\text{div } \mathbf{u}} = \text{div } \mathbf{U} = 0$$

$$\Rightarrow \text{continuity equation for the mean flow : } \text{div } \mathbf{U} = 0$$

- Next step, time average the momentum equation. This results in the Reynolds equations.

Turbulent flow - Reynolds equations

$$\begin{aligned} x - \text{momentum: } & \frac{\partial(\rho U)}{\partial t} + \text{div}(\rho U \mathbf{U}) = -\frac{\partial P}{\partial x} + \text{div}(\mu \text{ grad } U) + S_{Mx} \\ & + \left[-\frac{\partial(\overline{\rho u'^2})}{\partial x} - \frac{\partial(\overline{\rho u'v'})}{\partial y} - \frac{\partial(\overline{\rho u'w'})}{\partial z} \right] \end{aligned}$$

$$\begin{aligned} y - \text{momentum: } & \frac{\partial(\rho V)}{\partial t} + \text{div}(\rho V \mathbf{U}) = -\frac{\partial P}{\partial y} + \text{div}(\mu \text{ grad } V) + S_{My} \\ & + \left[-\frac{\partial(\overline{\rho u'v'})}{\partial x} - \frac{\partial(\overline{\rho v'^2})}{\partial y} - \frac{\partial(\overline{\rho v'w'})}{\partial z} \right] \end{aligned}$$

$$\begin{aligned} z - \text{momentum: } & \frac{\partial(\rho W)}{\partial t} + \text{div}(\rho W \mathbf{U}) = -\frac{\partial P}{\partial z} + \text{div}(\mu \text{ grad } W) + S_{Mz} \\ & + \left[-\frac{\partial(\overline{\rho u'w'})}{\partial x} - \frac{\partial(\overline{\rho v'w'})}{\partial y} - \frac{\partial(\overline{\rho w'^2})}{\partial z} \right] \end{aligned}$$

Reynolds stresses

- These equations contain an additional stress tensor. These are called the Reynolds stresses.

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} -\rho \overline{u'^2} & -\rho \overline{u'v'} & -\rho \overline{u'w'} \\ -\rho \overline{u'v'} & -\rho \overline{v'^2} & -\rho \overline{v'w'} \\ -\rho \overline{u'w'} & -\rho \overline{v'w'} & -\rho \overline{w'^2} \end{pmatrix}$$

- In turbulent flow, the Reynolds stresses are usually large compared to the viscous stresses.
- The normal stresses are always non-zero because they contain squared velocity fluctuations. The shear stresses would be zero if the fluctuations were statistically independent. However, they are correlated (amongst other reasons because of continuity) and the shear stresses are therefore usually also non-zero.

Turbulent flow - continuity and scalars

- Continuity: $\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{U}) = 0$

- Scalar transport equation:

$$\frac{\partial(\rho\Phi)}{\partial t} + \text{div}(\rho\Phi\mathbf{U}) = \text{div}(\Gamma_\Phi \text{grad } \Phi) + S_\Phi$$

$$+ \left[-\frac{\partial(\overline{\rho u' \phi'})}{\partial x} - \frac{\partial(\overline{\rho v' \phi'})}{\partial y} - \frac{\partial(\overline{\rho w' \phi'})}{\partial z} \right]$$

- Notes on density:

- Here ρ is the mean density.
- This form of the equations is suitable for flows where changes in the mean density are important, but the effect of density fluctuations on the mean flow is negligible.
- For flows with $T_i < 5\%$ this is up to Mach 5 and with $T_i < 20\%$ this is valid up to around Mach 1.

Closure modeling

- The time averaged equations now contain six additional unknowns in the momentum equations.
- Additional unknowns have also been introduced in the scalar equation.
- Turbulent flows are usually quite complex, and there are no simple formulae for these additional terms.
- The main task of turbulence modeling is to develop computational procedures of sufficient accuracy and generality for engineers to be able to accurately predict the Reynolds stresses and the scalar transport terms.
- This will then allow for the computation of the time averaged flow and scalar fields without having to calculate the actual flow fields over long time periods.