



STEADY

$$- \int_{\partial\Omega} \mu(\nabla w \cdot \check{n}) d\Gamma = - \int_{\Omega} \frac{dp}{dz} d\Omega$$

$$- \left[ \int_E \mu \frac{\partial w}{\partial x} d\Gamma - \int_W \mu \frac{\partial w}{\partial x} d\Gamma + \dots \right] = - \int_{\Omega} \frac{dp}{dz} d\Omega$$

$$- \left[ \mu_E \frac{W_{i+1,j} - W_{i,j}}{\Delta x(i)} \Delta y(j) + \dots \right] = - \int_{\Omega} \frac{dp}{dz} d\Omega$$

$$\alpha_{i,j} W_{i,j} + \alpha_{i+1,j} W_{i+1,j} + \alpha_{i-1,j} W_{i-1,j} + \dots = \beta_{i,j}$$

ONE EQUATION FOR EACH CELL (i,j)

STEADY

$$\alpha_{i,j}W_{i,j} + \alpha_{i+1,j}W_{i+1,j} + \alpha_{i-1,j}W_{i-1,j} + \dots = \beta_{i,j}$$

APPLYING MAPPING:  $i,j \rightarrow NJ^*(i-1) + j$

$$\begin{array}{llll} \alpha_{i,j} & \rightarrow & a_{I,I} & W_{i,j} & \rightarrow & W_I & \beta_{i,j} & \rightarrow & b_I \\ \alpha_{i+1,j} & \rightarrow & a_{I,J} & W_{i+1,j} & \rightarrow & W_J \\ \alpha_{i-1,j} & \rightarrow & a_{I,K} & W_{i-1,j} & \rightarrow & W_K \end{array}$$

$$a_{I,I}W_I + a_{I,J}W_J + a_{I,K}W_K = b_I$$

$$\begin{bmatrix}
 a_{1,1} & a_{1,2} & \dots & a_{1,NI*NJ} \\
 a_{2,1} & a_{2,2} & \dots & a_{2,NI*NJ} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{NI*NJ,1} & & & a_{NI*NJ,NI*NJ}
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 \vdots \\
 x_{NI*NJ}
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 \vdots \\
 b_{NI*NJ}
 \end{bmatrix}$$

(A)                      (x)                      (b)

REDUCING  
MATRIX  
BANDWIDTH



$$\left\{ \begin{array}{ll}
 NI < NJ: & NIJ = NI*(j-1) + i \\
 NJ < NI: & NIJ = NJ*(i-1) + j
 \end{array} \right.$$

STEADY

$$-\int_{\partial\Omega} \mu(\nabla w \cdot \check{n})d\Gamma = -\int_{\Omega} \frac{dp}{dz}d\Omega$$

$$A_f W = b_p$$

TRANSIENT

$$\int_{\Omega} \rho \frac{\partial w}{\partial t} d\Omega - \int_{\partial\Omega} \mu(\nabla w \cdot \check{n})d\Gamma = -\int_{\Omega} \frac{dp}{dz}d\Omega$$

$$\rho A_m \left( \frac{W^{n+1} - W^n}{\Delta t} \right) + \theta A_f W^{n+1} = b_p^{n+\theta} - (1 - \theta) A_f W^n$$

$$\left( \frac{\rho}{\Delta t} A_m + \theta A_f \right) W^{n+1} = \left( \frac{\rho}{\Delta t} A_m - (1 - \theta) A_f \right) W^n + b_p^{n+\theta}$$

# CONVECTION-DIFFUSION-REACTION

$$\int_{\Omega} \frac{\partial u}{\partial t} d\Omega + \int_{\partial\Omega} u(v \cdot \check{n}) d\Gamma + \int_{\Omega} r u d\Omega - \int_{\partial\Omega} k(\nabla u \cdot \check{n}) d\Gamma = \int_{\Omega} f d\Omega$$

$$\int_{\Omega} \left( \frac{\partial u}{\partial t} + r u \right) d\Omega + \int_{\partial\Omega} (-k \nabla u + u v) \cdot \check{n} d\Gamma = \int_{\Omega} f d\Omega$$

$$A_m \left( \frac{U^{n+1} - U^n}{\Delta t} \right) + \theta r A_m U^{n+1} + \theta A_f U^{n+1} = \\ b_f - (1 - \theta) r A_m U^n - (1 - \theta) A_f U^n$$

$$\left( \left( \frac{1}{\Delta t} + \theta r \right) A_m + \theta A_f \right) U^{n+1} = \\ \left( \left( \frac{1}{\Delta t} - (1 - \theta) r \right) A_m - (1 - \theta) A_f \right) U^n + b_f$$