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Short Communication

An allometric scaling law for understanding mammalian sleep

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An allometric scaling law is established for explaining sleep time per day which scales with brain mass. The theoretical prediction agrees well with Savage and West's experimental data.

Key words: Metabolic rate, white matter, gray matter, allometric scaling, sleep time.

INTRODUCTION

One of the most pervasive laws in biology is the allometric scaling, whereby a biological variable *Y* is related to the mass *M* of the organism by a power law (Leal da Silva et al., 2006; He and Zhang, 2004; He, 2005; He and Huang, 2006):

$$Y \propto M^{b} \quad (1)$$

where *b* is the so-called allometric exponent, and *Y* can be, for example, the heart rate, *f*, which scales as (West et al. 1997)

$$f \propto \frac{B}{M} \propto M^{-1/4} \quad (2)$$

There exist various theories arisen recently to explain various biological phenomena, such as allometrical method (West et al., 1997; Kuikka 2006), statistical method (Al-Suwaiyel et al., 2006) and E-infinity theory (El Naschie, 2006; El Naschie, 2007). In particular, using a blend of the methodology of allometrical scaling and E-infinity theory it was possible to solve various basic problems in biology (He, 2006a; He, 2006b; He, 2006c; He, 2007). In this short paper, a simple allometric scaling to explain mammalian sleep is suggested.

ALLOMETRIC METHOD

We consider the condition of slow wave sleep when both white and gray matters are completely inactive, in our publication (He, 2006b), we assume that

$$B_G \propto B_W , \quad (3)$$

where B_W is the basal metabolic rate of the white matter, B_G is the overall basal metabolic rate of the gray matter.
After simple derivation, we obtain (He, 2006b):

$$M_W \propto M_G^{6/5} , \quad (4)$$

where M_W is the volume of white matter , M_G is the volume of gray matter.
The prediction, (4), is very close to Zhang and Sejnowski's observation (Zhang and Sejnowski, 2000), which reads:

$$M_W \propto M_G^{1.23 \pm 0.01} .$$

In non-sleep period, the metabolically active brain has high specific resting metabolic rates when compared with the remaining less-active tissues, such as skeletal muscle, adipose tissue, bone and skin (He and Huang, 2006):

$$B_{brain} \propto M_{brain}^{0.81} \quad (5)$$

where M_{brain} is the mass of the brain, while the overall basal metabolic rate of the gray matter,

B_G, scales as (He, 2006b):

$$B_G \propto M_G^{4/5} \quad (6)$$

We have approximately:

$$M_{brain} \propto M_G \quad (7)$$

The scaling exponent, 0.81 (or 4/5), is remarkably higher than the universal value 0.75. During sleep we have:

$$B_G \propto B_W ,$$

the metabolic ratio mainly depends on that of the white matter.

We assume that the active time of the gray matter is t_A, and inactive time (sleep time) t_S, satisfying $t_A + t_s = 1$ day. We, therefore, have

$$t_A \propto \frac{M_G}{B_G} \quad (8)$$

$$t_S \propto \frac{M_W}{B_W} \quad (9)$$

Considering the scaling relationships, (3), (4) and (7), we have:

$$\frac{t_A}{t_S} \propto \frac{M_G}{M_W} \propto M_G^{-1/5} \propto M_{brain}^{-1/5} \quad (10)$$

Savage and West obtained experimentally the following equation (Savage and West, 2007)

$$\ln(t_A / t_S) = -0.21 \ln M_{brain} + 0.24 \quad (11)$$

Our scaling exponent, $-1/5$, is very closed to -0.21. Using the scaling relationship, (6), we predict that:

$$\frac{t_A}{t_S} \propto M_{brain}^{-1/5} \propto M^{-0.162} \quad (12)$$

which agrees very well with Savage and West's result, which reads (Savage and West 2007):

$$\ln(t_A / t_S) = -0.16 \ln M + 0.18 . \quad (13)$$

Conclusion

We give a very simple allometric approach to explanation of mammalian sleep, the scaling exponent agrees well with Savage and West's experimental data.

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